October 1977
by Jack Riedel

3ϕ Model, Q, Transmitter, and Other Things

First, let us review the purposes of these notes. They are to document for posterity what is going on in these design phases, and to impart information to others who, while not actively working on the RF system, are still a member of the team that is going to get this superconducting cyclotron running. Any or all of these people may be called upon, in time, to rescue the project from failure by contributing to the RF system, without which no acceleration of beam is possible! This is said because, firstly, this is a very very difficult RF system to design and build and secondly, without the cooperation of almost everyone, it will be impossible to make work.

3ϕ High Q Model

Scott Francis has done very well in debugging the electronics and mechanics of the 3ϕ Model. Suddenly it occurs to me that various people may not know what the model is, and what its purpose is. Therefore, we present Fig. 1, a mechanical view of the model; or you can walk up behind the magnet power supply and view it.

Fig. 2 is a schematic diagram of the equivalent circuit of the model. There were two reasons for building this expensive model. The first was political. Since the only people (W. Smith and K. MacKenzie of LRL) who had previously tried to make a 3ϕ rf system work had asserted that it was impossible to do so unless the coupling between the three dees was neutralized, it was natural that people would be skeptical when I said that my measurements on a crude model, and calculations with ECAP, convinced me that it could be done. This point was crucial to the decision about the upper frequency limit of the RF system, because if 3ϕ operation was not possible, the rf frequencies would have to be three times higher to accelerate the same particles.

The second reason for building the model was that it would provide us with an instrument, a simulated three dee hi Q cyclotron rf system, that would be appropriate for us to install and debug all the low level electronics that would eventually be used on the cyclotron. Non RF people probably fail to realize that most of the man hours required to get a functional rf system go into building and debugging the low level electronics for control and regulation.

Fig. 2 is a schematic of the essential electronics used for Francis' tests to substantiate the claim that 3ϕ operation is not only possible, but well behaved and easy to achieve. The results agreed completely with the calculations from ECAP. The high gain phase loops guaranteed that the phases of the 3 dees were 120° apart, and manual adjustment of the 3 drivers could modify the relative amplitudes with about a 4 to 1 favoring of the one selected for modification. It is picayune to suggest that there will be any problem with servoe amplitude control. The demonstration that I witnessed completely convinced me that 3ϕ operation will impose no problems, and further tests on this are unnecessary.

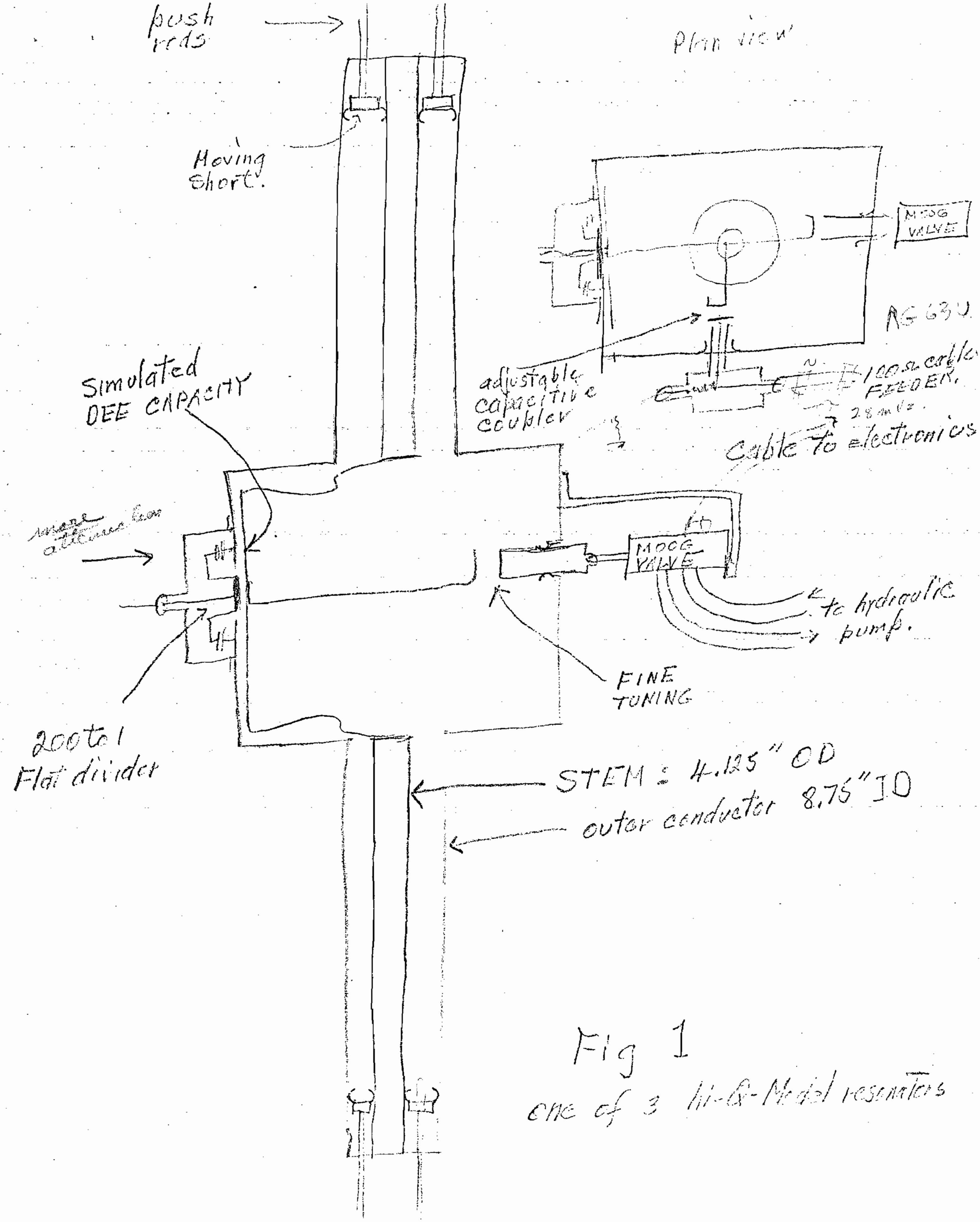


Fig 1

one of 3 Hi-Q-Model resonators

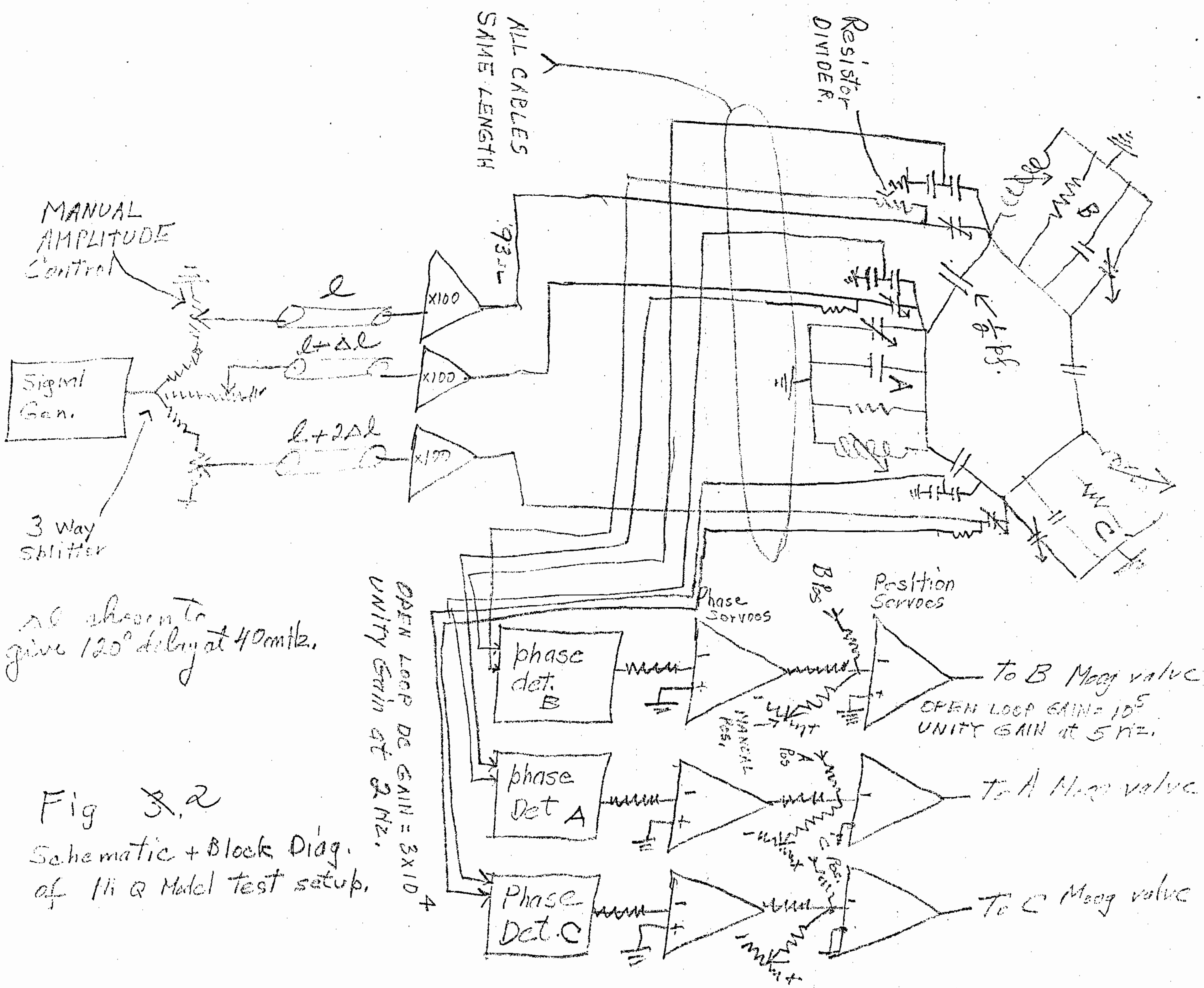


Fig ~~3~~ 2
Schematic + Block Diag.
of Hi Q Model test setup.

Q

Q is a dimensionless number, an abbreviation for "quality factor". It is used to describe a parameter about an oscillating system and is exactly defined as

$$Q = \frac{2\pi \text{ x peak energy stored per cycle}}{\text{energy lost per cycle}}$$

equally appropriate for pendulums, flywheels, Earth-moon systems, tuning forks, and RLC resonators. In the case of the moon, Q is a large negative number because of tidal forces on the Earth. Here on Earth, Q is almost always positive and small, like my monthly bank balance, a very oscillating system.

The reason for interest in Q is that it is an intrinsically measurable quantity that can tell us how much power it will take to make an oscillator oscillate at a given amplitude.

Now Q. Kerns of FNAL asserted that the Q of his resonators was lower by 30% from what he had calculated they should be, based on the skin resistance of copper at his frequencies. So we used one of the hi Q model stems to measure its Q and compare the results with the calculated value.

Now it turns out that it is a little tricky to calculate the Q of a complex resonator, and not so simple to even calculate the Q of a $\lambda/4$ transmission line. If you look up in the literature you will find discrepancies of π or 2 in the published results. Further, in trying to make a quickie calculation of the Q of our $\lambda/4$ resonator, I found such discrepancies apt to occur for different methods of making the calculation. This caused me to question the results of my program for calculating the power requirements of the final rf system.

Therefore I went back to basics and tried to think it all out clearly. The results of several hours of doing this appear in Appendix I of this note: "On Q and related matters". My final formula for the Q of a capacitive loaded $\lambda/4$ transmission line resonator is:

$$Q = \frac{2\pi\sqrt{F}\, Z_o}{3 \times 10^8 k} \times G(c) \quad \text{where } G(c) = 1 \text{ for a } \lambda/4 \text{ line}$$

where $k = 2.63 \times 10^{-7} \left(\frac{A + B}{\pi AB}\right)$ all MKS units

A = outer diam, B = inner diameter

C = capacitor

O_o = atn (wcZ_o)

Z_o = 138 log A/B

I swear that this equation is correct! But wouldn't mind corroboration.

It hurts to admit that about a year ago I asserted that the Q and power for a capacitive loaded line were the same as for an open line. But notice that this is untrue: adding a lumped infinite Q capacitor to a line increases the Q and lowers the power.

Naturally I rechecked my program and, lo, discovered that it was correct! After breathing a deep sigh of relief I now understand why my calculated Q's were higher, and the power lower than given for simple, incorrect formulae.

Although Francis has not completed making accurate measurements, and getting the two different measurement methods to agree, it appears that his $\frac{\Delta F}{F}$ measurement of Q = 5780 is only 17% lower than any calculated value of 6954. For the present, let us accept this as correct, and the actual power will be some 20% greater than that published in RF Note #17.

Enuf of Q.

Transmitter

RF note #13 (8-20-77) was a first look at the transmitter design. Since then the top frequency has been lowered to 32.5 mHz (from 60) and we have decided to arrange it such that the bottom frequency for the transmitter will be 7.5 mHz. For operation up to 32.5 mHz we no longer need to use the 4CW100000J tube but (cont. on next page)

can stick with the 4CW100,000D tube, which is the one used in the present cyclotron. Now one of the problems with a transmitter tunable over a 4 to 1 frequency range is how to find the B+ in. Without taking the time or space here to describe the standard ways of doing this, and discussing the demerits of these ways, I will simply present the method we have chosen. Figure 3 is a schematic of the anode circuit.

Explanatory notes for Figure 3 (which is on next page) are below.

point A is the anode. V_A = 20,000-18,000 Sincot
C1 is the blocker = 2000 pf.
C_0 = is the anode cap. = 70 pf.
$I1 = 18000 \times wC_0 \times \cos(wt) = 257 \cos(wt)$
for F = 32.5 mHz. or 59.4 Amp @ 7.5 mc

C2 is a 2000 pf barium titanate capacitor

$V_A - V_B = I1/wC1 = 630$ volts.
Let L1 = 5 μH then

$$I2 = \frac{630}{wL1} = \begin{matrix} .6 \text{ amps at } 32.5 \text{ mHz} \\ 2.6 \text{ amps at } 7.5 \text{ mHz} \end{matrix}$$

$$V_A - V_C = I1/wC2 = \begin{matrix} 22.4 \text{ volts at } 7.5 \text{ mHz} \\ = 11.6 \text{ volts at } 32.5 \text{ mHz} \end{matrix}$$?

C2, C3 and C4 are Sprague Barium Titanate capacitors rated at 30 KV dc. They have a Q of 60 and thus will dissipate a maximum of 1 watt.

This voltage, $V_A - V_C$, can be considered as a very low impedance generator feeding the transmission line consisting of the RG 19V as the inner conductor and the 4" ID center conductor of the anode resonator. From point B to point D there is a $\frac{dd}{dt}$ such that $V_D = V_A - V_C$ if point D looks into Z_0, the impedance of the 24' line. We will make Z_0 = 50 ohms by leaving the jacket on it and connecting this jacket at both ends to point B and ground. The RG 19V cable to the power supply will be terminated there by the circuit shown.

Now if R1 = 100 ohms, then at a frequency such that there will be a resonance in the cables, there can be a standing wave in the cables, but the rf voltage can nowhere be greater than 2 times $V_A - V_C$, or 50 volts. Thus, at no time will the dissipation in R1 or R2 exceed 50 watts. So R1 and R2 are globars rated at 100 volts and 50 watts. Without R1 and R2, and with Q's of 600 for the cable, we could have voltages of 600 x 25 = 15000V in the cable and it would burn up. Without L1 and C2, R1 and R2 would have to be rated at 5KW.

Figure 4 is a sketch showing a physical arrangement for which, hopefully, Fig. 3 is the equivalent circuit. Everything is very straightforward about this circuit and easily calculatable except for L1. So we start with 3 turns and when the first transmitter is built we may need to modify L1. This will be done by driving point A with a generator and measuring V_D and V_E and adjusting L1 so that nothing bad happens over the frequency range 7.5 to 160 mHz,

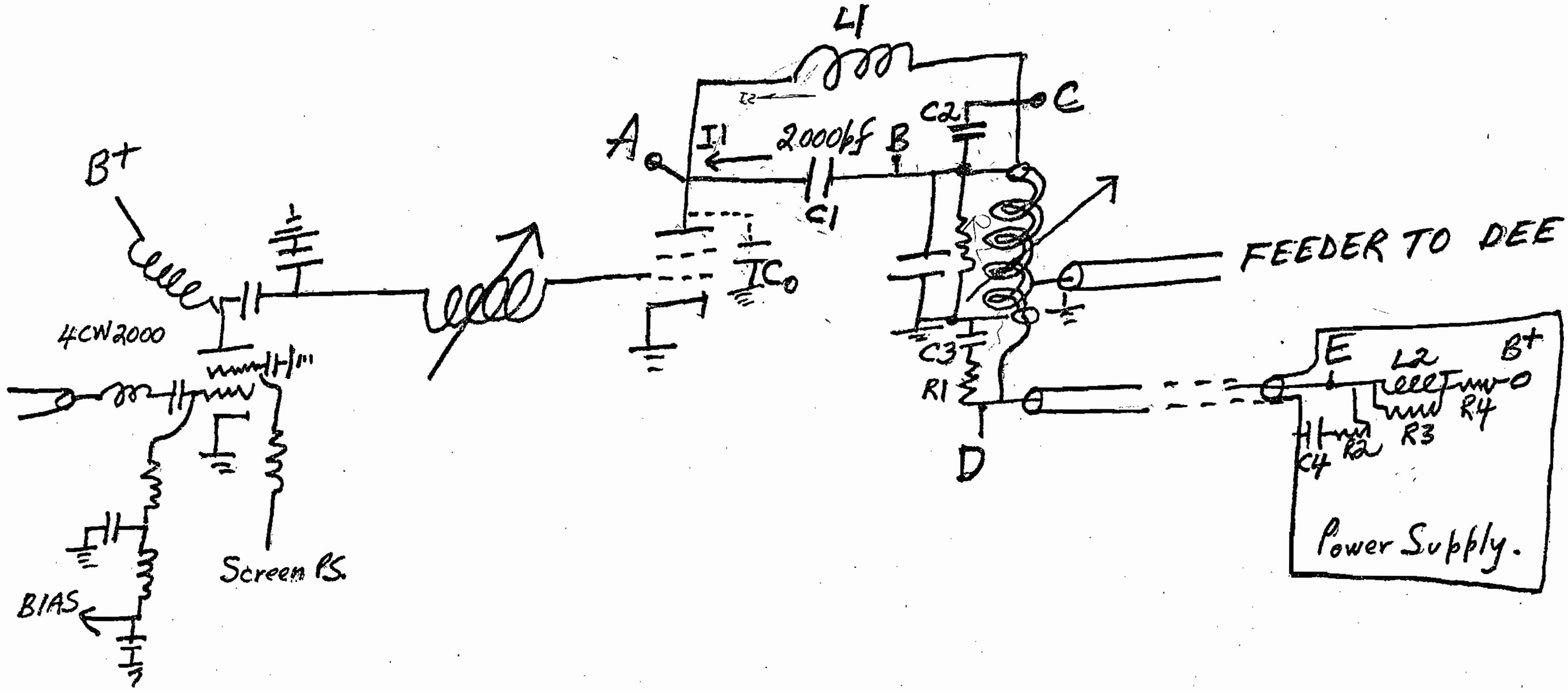


FIG 3
Schematic of Transmitter.

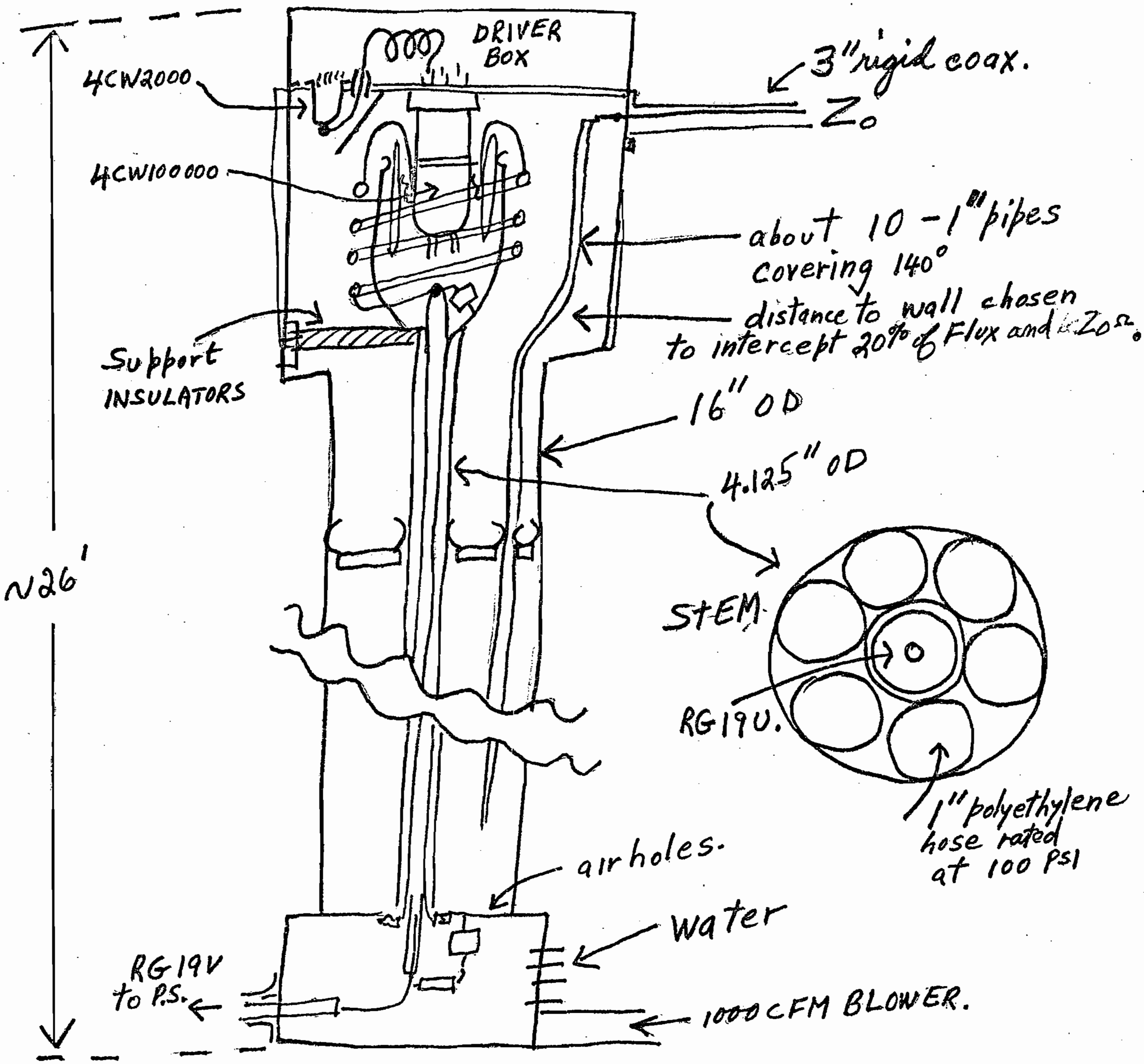


FIG. 4

SKETCH OF TRANSMITTER

i.e. up to the 5th harmonic of 32.5 mHz. Also, grid dipping L1 should result in a resonance at 1.2 MHz.

Calculations

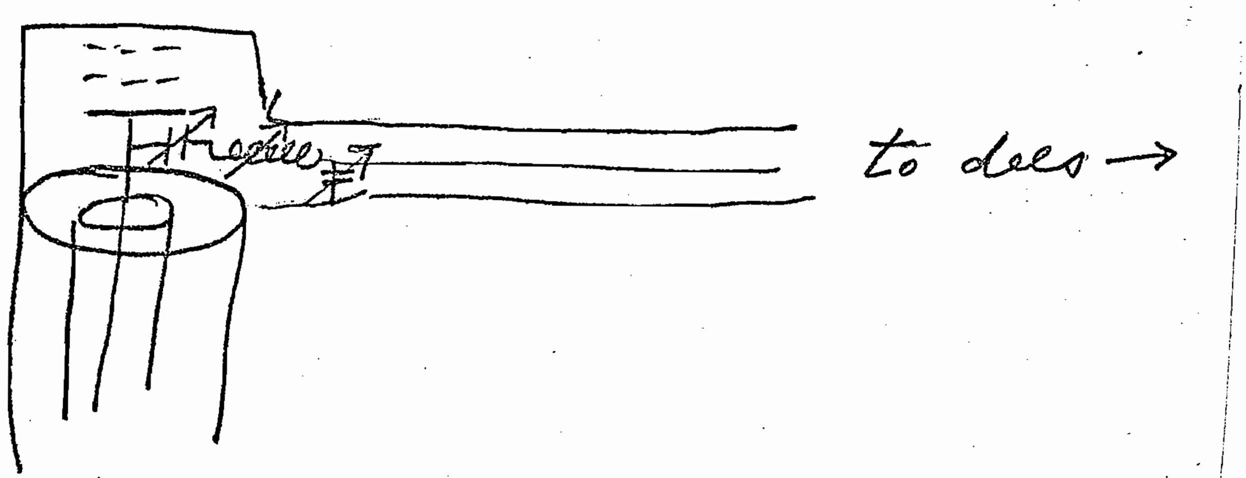
Program TRANS calculates this system and the results are presented in Table I. An endeavor was made to reduce the maximum length of the line by adding capacity at point B, but this made it difficult to achieve 32.5 mHz. Since the maximum possible length is about 24 ft., it means that we will have to plug in vacuum capacitors at point B to achieve frequencies lower than 10 mHz. This is consistent with the dee tuners which also will have to have capacitors added at the insulator to get below 10 mHz.

Moving Short

The moving short will be the same as for the dee stem, except for the larger outer diameter, the one inch holes thru which the output lines penetrate, and the fact that no water cooling will be necessary. The mover, using side coupled bicycle chains will be identical with the dee stem mover.

Output

To drive the 50 ohm line to the dees (75Ω is a better choice) we need to transform the plate rf voltage by a factor of 1/5. Conventionally, this is done with a tuned π or T network as in the diagram below.



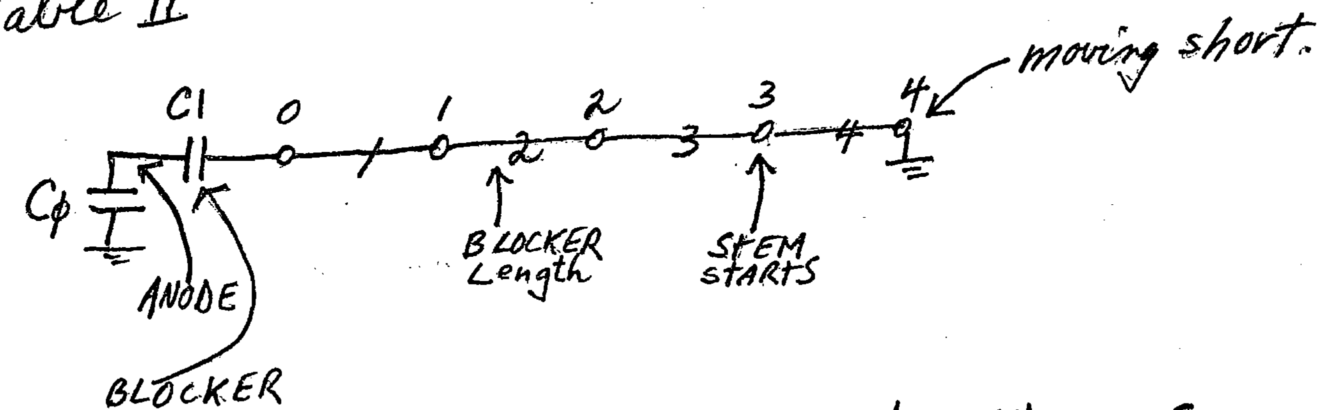
Obviously a very clumsy arrangement, fraught with parasitic mode traps, especially so over such a large frequency range as we have.

It is also possible to design a non-tuned system, basically an exponential line which acts as a transformer over a certain bandwidth. For a 4 to 1 range this would require at least 4! or 24 elements and be even more likely to cause trouble with parasitic

F	ΔL	L	W	I_p	Q	R_s	C_{EQ}
MHz	INCH	INCH	watts	RMS A	--	KΩ	Pf
10	204	236	340	170	5300	250	340
15	107	139	290	180	6700	290	243
20	59	91	260	190	8300	330	201
25	32	64	220	200	10 000	370	180
30	14	77	200	220	13000	414	166
35	3	35	170	240	17000	485	158

TABLE I

where the symbols are the same as in RF Note #17 also, as in RF note #17, we can present the geometry and Table II



N	ΔL	L	Z_ϕ	A	B	W_i	W_o	C
								70
0	0	0	—	—	–	—	—	
1	6	6	96.5	30	6	25	5	0
2	22	28	83.1	30	7.5	95	20	0
3	4	32	120	30	4.12	30	4	0
4	—*	—*	79.3	15.5	4.12	300	80	0

TABLE II

* see TABLE I

modes. So we use the loop coupling method, with the flux enclosed by the loop always 1/5 of the total flux. This can be accomplished automatically over the frequency range by placing the holes in the moving short at the correct radius. Also, so as not to have to worry about the self inductance of this loop, we make it everywhere have a Z_0 of 50 ohms. Or 75. Yes, 75 is better, for a variety of reasons:

1. To deliver a given amount of power with a fixed OD of conductor, 66.7 ohms results in the minimum line loss, and 75Ω consumes 25% less power than 50Ω.
2. The holes in the anulus don't have to be so close to the outer conductor.
3. The dee coupling capacitor can be smaller by the ratio 50/75.
4. The losses in the vacuum part of the feeder will be less, a distinct advantage.

The only disadvantage is that 75Ω line is not made in semi-flexible form. But Prodelin makes 75 ohm rigid coax, and it is cheaper than the semi-flexible kind, and also now we can use 3-inch rigid coax instead of 4", at a considerable saving.

Plans

The design of the dee, dee stem, insulator, moving short etc. is in such a state that detailed drawings should be made forthwith and one dee plus its two stems etc. should be fabricated so that full power and voltage tests can be made. The same applies to the coupling capacitor and feeder. A decision must be made as to whether to construct a test vacuum chamber to house the dee or to make the test in the magnet.

Within the week the power supply specifications will be completed and we should go out for bids on this as soon thereafter as possible.

Detailed drawings of the transmitter should also be made and the first one constructed.

To accompany the power supply specifications I propose to write a diatribe titled, "On power supplies for hi power rf systems". Since no one was willing to bid on the special purpose synthesizer we will have to build the special output circuits ourselves. This means we don't have to buy a synthesizer, but it does mean six man months of design, development and construction by a good circuit man. We should start on this forthwith.

APPENDIX 1 TO RF NOTE 18

On Q and Related Matters

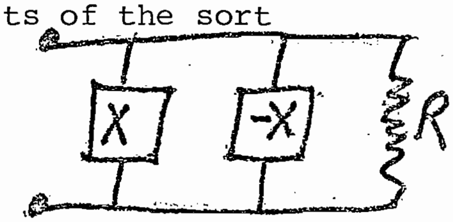
Q is a parameter that describes the relationship between stored and lost energy in a system which is executing oscillations. This system is defined as a system where energy oscillates between the kinetic and potential forms, and the most fundamental definition of Q is

$$Q = 2\pi \frac{\text{peak energy stored per cycle}}{\text{energy lost per cycle}}$$

which results in

$$Q = \frac{E}{W} = \frac{\text{circulating energy}}{\text{power consumed}}$$

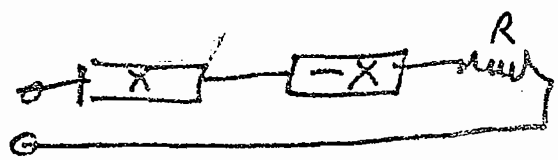
for lumped circuits of the sort



$$Q = \frac{E}{W} = \frac{V_0^2/X}{V_0^2/R} = \frac{R}{X} \quad \text{and if } X = \omega L \text{ or } \frac{1}{\omega C}$$

$$Q = R\omega C = \frac{R}{\omega L} \quad \text{for } \omega^2 = 1/LC$$

and for lumped circuits of the sort



$$Q = \frac{X}{R} = \frac{\omega L}{R} = \frac{1}{\omega RC}$$

For this latter configuration (series LRC), if a constant voltage is impressed across the terminals at a F such that

$$\delta = \frac{\Delta F}{F_\phi}$$

where F_ϕ is the resonant frequency we can write down the equations for the "universal resonance" curve

$$\frac{I}{I_0} = \frac{\text{current at } F}{\text{current at } F_0} = \frac{1}{1 + \delta + jQ\delta\left(\frac{2+\delta}{1+\delta}\right)}$$

when for off resonance

$$\frac{I}{I_0} = \frac{1}{Q\gamma\left(1-\frac{1}{\gamma^2}\right)} \quad \text{where } \gamma = \frac{F}{F_\phi}$$

$$\text{and } \tan\theta = Q\left(1-\frac{1}{\gamma^2}\right)$$

and for parallel resonant circuits the same sort of equations hold, except we substitute the ratio of impedances for the current ratios.

One consequence of these equations is that we can write down a simple rule for measuring Q with the $\frac{\Delta F}{F}$ method. For a parallel circuit, using a <u>constant current</u> drive and measuring V/V_O where V_O is the voltage at resonance, then

$$\frac{V}{V_0} = \frac{1}{\sqrt{2}} \text{ when } \frac{\Delta F}{F} = \frac{1}{2Q} \text{ and } \theta = 45^\circ$$

$$\frac{V}{V_0} = .44 \text{ when } \frac{\Delta F}{F} = \frac{1}{Q} \text{ and } \theta = 63\tfrac{1}{2}^\circ$$

For these $\Delta F/F$ measurement of Q it is essential that the drive and monitor connections do not load the resonator. Now for a series LRC circuit of $Q=WL/R$, a good approximation for $Q \gg 10$ of the current after the voltage source has been removed is

$$i_{(t)} = i_0 e^{-\frac{\pi F}{Q}t} \sin \omega t$$

showing that $\frac{i}{i_0}$ or $\frac{V}{V_0} = \frac{1}{e}$ when $t = \frac{Q}{\pi F}$ seconds

or when $\frac{Q}{\pi}$ cycles have elapsed.

Thus Q can be measured by the decrement method by cutting off the drive to a resonator.

For a high Q resonator of $Q \sim 5000$ it is very important, for accuracy, that the monitor not load down the resonator, and that the drive current be completely removed. The best way to assure that these criteria are true is to plot t, the time constant as a function of decreasing the monitor coupling and the drive coupling by factors of two until t does not change.

Calculation of Q

To calculate Q for a resonator consisting of lumped constants plus distributed parameters such as transmission lines one goes back to the fundamental definition,

$$Q = \frac{E}{W} = \frac{\text{total circulating energy}}{\text{total power consumed}}$$

To do this one calculates E and W independently. First one must solve the network equations that will permit one to write down, for each element V and I of t and x, then

$$E = \sum \Delta (V \times I)$$

where V and I are rms values, and

$$W = \sum \Delta I^2 R$$

We will now detail how to do this.

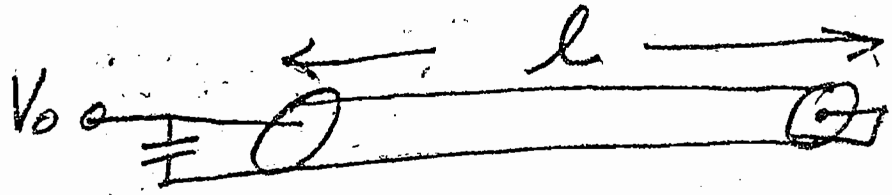
A. The Circulating Energy

For distributed lines there are three ways of calculating E for a V_O:

1. $E = \sum \Delta (V^2_{rms}\, \omega C) = \sum \omega \times$ peak capacitive stored energy
2. $E = \sum \Delta (I^2_{rms}\, \omega L) = \sum \omega \times$ peak inductive stored energy
3. $E = \sum \Delta (\vec{I}_{rms} \times \vec{V}_{rms})$ cross product.

We will show that method 1 and 3 are equivalent.

First we will tackle a capacitive loaded $\lambda/4$ transmission line resonator:



A = outer diam, meters

B = inner diam, meters

ℓ = length of line

C = capacity loading, Farads.

F = the frequency

V_o = rms voltage at x = 0

Θ_o = phase of incident wave entering the line.

$Z_o = \sqrt{\frac{L_e}{C_e}} = 138 \log_{10} \frac{A}{B}$ the characteristic impedance of the line.

$I_c = V_o \omega C$ and

$E1 = \omega C V_o^2$ by method 1 or

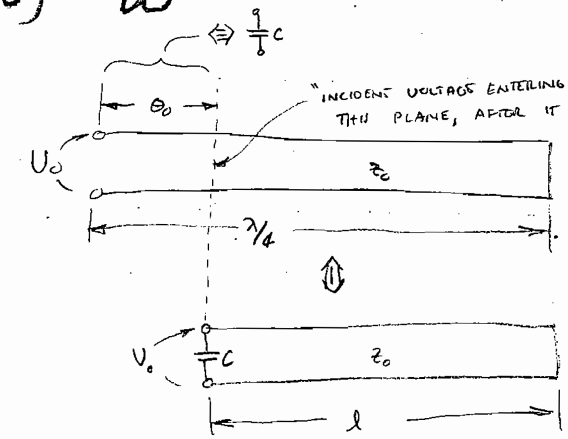
$E1 = V \times I = V_o^2 \omega C$ since V & I are orthogonal by method 3.

Now the phase angle of the incident voltage entering the lines is

LIKE THE ELECTRICAL LENGTH OF THE TRANS. LINE THE C REPLACES

$$\theta_o = \operatorname{atn} \frac{I Z_o}{V} = \operatorname{atn}(\omega C Z_o)$$

$$\ell = (\pi/2 - \theta_o) \times \frac{c}{\omega} \quad \text{where } c = \text{velocity in cable.}$$



"INCIDENT VOLTAGE ENTERING LINE" REFERS TO THE VOLTAGE AT THIS PLANE, AFTER IT HAS TRAVELED DOWN THE "LENGTH OF LINE" THE CAPACITOR REPLACES

$Z = j Z_o \cot \theta_o$

$Z = 1/j\omega C = \frac{V_o}{I_c}$

$\cot \theta_o = \frac{1}{\omega C Z_o}$

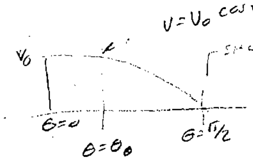
$\tan \theta_o = \omega C Z_o$

$\theta_o = \tan^{-1}(\omega C Z_o)$

$= \tan^{-1}\left(\frac{I_c Z_o}{V_o}\right)$

In the line

$$V_\theta = \frac{V_0}{\cos\theta_0}\cos\theta \qquad V_\theta = V_0 \cos\theta_0 \cos\theta \ ???$$



$$I_\theta = \frac{V_0}{Z_0\cos\theta_0}\sin\theta \qquad I_\theta = \frac{V_0}{Z_0}\cos\theta_0 \sin\theta \ ???$$

$$E2 = \sum \Delta(V^2 \omega C_0) \qquad \text{now } \Delta C = \frac{\Delta x}{3E8\, Z_0}$$

$$\text{but } \Delta x = \frac{3E8\,\Delta\theta}{\omega}$$

$$\text{so } \Delta C = \frac{\Delta\theta}{\omega Z_0}$$

$$E2 = \frac{V_0^2}{Z_0\cos^2\theta_0}\int_{\theta_0}^{\pi/2}\cos^2\theta\, d\theta = \frac{V_0^2}{2Z_0\cos^2\theta_0}\left(\frac{\pi}{2} - \theta_0 + \frac{\sin 2\theta_0}{2}\right)$$

$$E = E1 + E2 = V_0^2\left(\omega C + \frac{1}{2Z_0\cos^2\theta_0}\left(\frac{\pi}{2} - \theta_0 + \frac{\sin 2\theta_0}{2}\right)\right)$$

and

$$W = \sum \Delta I_x^2 R_x = \frac{3E8\, R_x V_0^2}{\omega Z_0^2 \cos^2\theta}\int_{\theta_0}^{\pi/2}\sin^2\theta\, d\theta \qquad 4.5$$

$$= \frac{k\cdot 3E8\, V_0^2}{4\pi\sqrt{F}\, Z_0^2\cos^2\theta_0}\left(\frac{\pi}{2} - \theta_0 - \frac{\sin 2\theta_0}{2}\right) \qquad k = \frac{2R_x}{\sqrt{F}}$$

where $k = 2.63\times 10^{-7}\,\frac{A+B}{\pi AB}$, $A+B$ in meters

for $C = 0 \rightarrow \theta_0 = 0$ we have

$$E = \frac{\pi V_0^2}{4Z_0}, \qquad W = \frac{3E8 \times k V_0^2 \times \pi}{8\pi Z_0^2\sqrt{F}}$$

$$Q = \frac{2\pi Z_0\sqrt{F}}{3\times 10^8\, k}$$ --- the Q of a $\lambda/4$ line

and for a capacitive loaded line we can express the Q as

$$Q = Q_0 \left(\frac{\frac{\pi}{2} - \theta_0 + \frac{\sin 2\theta_0}{2} + \frac{Z_0}{X_c} \cos^2 \theta_0}{\frac{\pi}{2} - \theta_0 - \frac{\sin 2\theta_0}{2}} \right)$$

where $Q_0 = \frac{2\pi Z_0 \sqrt{F}}{3 \times 10^8 k}$ and $X_c = \frac{1}{2\pi F}$

it is easy to see that $Q > Q_0$ for the same k and putting in the numbers for $X_c = Z_0$ making $\theta_0 = \pi/4$ we have

$Q = 13.5\, Q_0$.

Now for an example:

$\theta_0 = 0$ ($\lambda/4$ line)

$A = 8.75''$

$B = 4.125''$

$Z_0 = 45.07$

$\frac{A+B}{\pi AB} = 4.47$/meters

$k = 1.176 \times 10^{-6}$, $R_x = 1.018 \times 10^{-2}\ \Omega$ / meter

$Q = 6954$

Scott Francis measured $Q = 5780$

error $= -17\%$

Also, at 5 mHz $Q = 1795$ which checks with my program calculation

Note I have neglected the losses in the fingers and anulus & estimate these to be 5% of W, so my calculated Q will be reduced by 5% making the error only 12%.