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by JR.

Stem rf losses for DSWI

1. Maximum power density occurs at the highest frequency at the short position. The computer printout gives ~~...~~ 2600 amps at 30 mHz and an R_L of 5.5×10^{-3} Ω/meter for the combined skin resistance. Digging into the Program we find that

$$R_L = 2.63 \times 10^{-7} \sqrt{F} \left(\frac{A+B}{\pi AB} \right) = R_A + R_B$$

where A & B are the outer and inner diameters in meters, and R_A & R_B are the resistance per meter of the outer and inner walls respectively. This can be untangled to render

$$R_A = \frac{R_L}{C+1}, \quad R_B = R_L - R_A \quad \text{where } C = \frac{A}{B}$$

$$\frac{A}{B} = \frac{16}{4.12} = 3.88 \text{ so } R_A = 1.127 \times 10^{-3} \ \Omega/m$$

$$R_B = 4.37 \times 10^{-3} \ \Omega/m.$$

Thus the power densities are

$$W_A/\text{sq in} = I^2 R_L \times \frac{1}{39.37 \times \pi A} = \underline{3.85} \text{ watts/sq in.}$$

W_B/sq in = <u>58</u> watts/sq in.

However, I now remember that because we have a hexagon rather than a circle the currents are concentrated in the middle of the flats, so <u>probably</u> the ~~po~~ peak power density on the outer conductor will be about 8 watts/in^2

2. Total stem power, inner & outer.

at 30 mHz the stem is only 3 inches long, so the total power for inner and outer is only 2600 watts: 2300 watts on inner conductor and 300 watts on outer conductor. The total stem power will be maximum at low frequencies.

at 10 mHz, total power is 16000 W on a 106" stem; 14 KW on inner conductor and 2 KW on outer conductor.

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January 19, 1978

RESISTIVE PAPER FIELD MEASUREMENT METHODS AND INSULATOR CORONA RING DESIGN

I. General Considerations

When a voltage is applied to a system of conductors, the electrical potential between them is the solution of Laplaces equation for those particular boundary conditions. Sometimes in RF work we have to deal with large voltage, and hence large electric fields, and it becomes very desireable to know just how large these fields are, as sparking problems can become most annoying.

Computers can numerically reach a solution to Laplaces equation, but with complicated geometries of conductors the boundary conditions that must be satisfied are tedious to specify to the computer, and such programs require a lot of memory space.

An alternative approach to solving L. eq. is using resistive paper. By using silver conductive paint one can designate the desired equipotential surfaces simply by painting them on the paper. Then when a voltage is applied to the desired surfaces, a small current flows through the paper, producing a continuously varying electrical potential on the surface of the paper that automatically satisfies Laplaces eqn. Thus by mapping equipotentials and determining the gradient of the potential at points of interest, we may "measure" the electric fields at those points.

Of course, the paper is only a two-dimensional surface, and care must be taken when attempting to model a 3-dimensional configuration on it. Ultimately we must make use of some symmetry of the 3-dimensional system in order to accurately model it on paper. A particularly simple system of conductors to model on paper is that comprised of two infinite parallel planes are themselves infinite planes, parallel to and in between the two conductors. If we look at a cross-section of this system by intersecting it by a plane perpendicular to it, we have a pair of infinite parallel lines, with equipotentials being also infinite lines. If we paint two conducting infinite parallel lines on resistive paper, and apply a voltage between them, we get the same result as the cross section of the 3-dimensional system. Another simple system is one of two infinite parallel strips as in a parallel plate transmission line. In this case, we notice that a cross section of this is just two parallel finite lines.

As you have already guessed, this cross section is our 2-dimensional model on resistive paper, and theoretically gives exact results, as long as the actual transmission line is infinitely long. If it is not infinite, then we have edge effects, and the fields at the ends are not quite what the paper model says. However, if the length of the line is much larger than the separation and width of the strips, then not much error is introduced. In quite the same fashion, a coaxial transmission line may be represented in two dimensions, by its cross-section perpendicular to its axis of cylindrical symmetry. All these representations give measurements of potentials and gradients that are theoretically exact in the limit that the planes or lines extend infinitely in the direction perpendicular of the plane of the cross-sections that are their 2-dimensional representations. I have made drawings of these systems and corresponding 2-dimensional models, these being the first page of illustrations.

In some cases, however, we have no such symmetry to enable us to construct an accurate 2-dimensional model of a 3-dimensional system. One such case is that of a section of coaxial transmission line, whose length is small compared to the inner and outer radii. In this instance edge effects are most significant, and we cannot use the cross section that is perpendicular to the axis of symmetry. We can however use the cross section produced by intersecting the 3-dimensional system with a plane which contains the axis of symmetry. Now, though, we must realize that this cross-section cannot give us the proper results as we can understand by considering the case of an infinite coaxial transmission line, this time represented by its cross-section that contains its axis of symmetry. Let us, for arguments sake, call the inner conductors (outer) radius a, and the outer conductors (inner) radius b. Then the cross-section of this transmission line is just four infinite parallel lines, parallel to, say, the Y axis, and intersecting the X axis at X=-b, -a, +a, and +b. Since we are at present interested in the electric field between the two conductors (that is in the regions $-b<x<a$ and $a<x<b$) and it doesn't matter which side we consider, let us consider only one half of this cross-section, with the inner conductor at X=a the outer at X=b. Now the 2 dimensional representation of our coaxial transmission line is precisely that of our system of two infinite parallel planes, separated by the distance d=b-a.

Drawing these two parallel lines on resistive paper, and painting them in with conductive paint to produce the desired equipotential bounding conditions, and applying a voltage V across them we see as is no surprise, a uniform gradient (-E field) of V/d, which is the precise electric field for a system of infinite parallel plane conductors. It is a well known result of Gauss' law, however, that the electric field between two coaxial cylindrical conductors varies as 1/r, r being the distance from the common axis. Thus one may compute the field between the conductors by saying E(r) =K/r, K being same constant that depends upon the voltage between the two conductors (the 1/r dependence being independent of V, as is the whole (dense) family of equipotential curves, both of these being a consequence of the unique solution of Laplaces equation for the coaxial geometry under consideration). With the knowledge that $V=-\int_a^b E\,ds$ and E=K/r, we get $V=-K\int_a^b 1/r\,dr$, using appropriate symmetry, and find V=-K ln b/a, the minus sign indicating that the direction of integration went from the larger to the (relatively) smaller potential. Thus K=V/(ln b/a), (dropping the sign) and $|E(r)|=K/r$, the direction of the electric field being a trivial matter.

All this is quite elementary, but helps to illustrate the nature of an intrinsic error that may be incurred when one wants to use a two dimensional model of a 3-dimensional system. If the reader is not already asleep, I propose to explain how these concepts are used in designing parts of the new RF system, and present some of the results of these investigations.

II. Corona Ring Sparking Test

When my first efforts appeared to say that our original design of the insulator corona ring would not allow us to put 100 KV on the dees, it was decided to put this resistive paper technique to the test. Consequently a simulation of our corona ring was machined out of aluminum, a disk about 3 inches thick, 18 inches in diameter, with edges rounded with curves of radii 1/2 inch and 3/8 inch. (Illustration in back). This disk was then centered inside a steel drum of diameter 22.5 inches, the idea being to see how much voltage could be put on the disk (relative to the drum) without causing that nasty sparking. Meanwhile, I was to paint up a resistive paper model of this and determine what this critical voltage is to, shall we say, calibrate this paper technique. The second page of drawings shows this test setup and 2-dimensional model. The seven points on the edge of the disk labeled A-G are the points I "measured" the gradients at. Putting 10 volts on the disk, with the drum at ground, located the 9.50 volt equipotential and measured the distance d of this equipotential from the disk at the points A-G. Since $|E| = \vec{\nabla} V$, or V/d if d is relatively small, I get an estimate of the electric field. Since we want the ring to hold about 100 KV, and 5% of 100 KV is, of course, 5 KV, my zeroth order estimate of the gradients are 5/d KV/in, with d in inches. But now I must invoke the earlier argument about the error incurred when modeling a cylindrical system as I have in this test. With a coaxial transmission line of inner radius a=9" and outer radius b=11.25", and a voltage of 100 KV across them, we find the gradient at the surface of the inner conductor to be 49.8 KV/in, since $E(a) = K/a$, $K=V/\ln b/a$, or E (a)=V/a ln b/a = 100 KV/9" ln (11.25/9) = 49.79 KV/in. But in the 2-dimensional model of the coaxial transmission line that consists of 2 parallel lines (as in illustration) we measure with 100 KV across a separation of 2.25 inches, a gradient of 100 KV/ 2.25 in, or 44.44 KV/in. Thus at the radius a, this technique underestimates the electric field by about 12%. So my first order estimates of the gradients are the zeroth order ones multiplied by 49.79/44.44 = 1.120. This fudge factor I call the cylindrical correction factor, and is different for different radii, and different geometries. The results of these measurements are given in the table below, assuming 100 KV on the disk.

Point	Distance to 95% equipotential (in)	Zeroth order field (KV/in)	First order Fiel (KV/in)
A	.058	86.2	96.5
B	.055	90.9	101.8
C	.050	100.0	112.0
D	.085	58.0	65.0
E	.062	80.6	90.3
F	.067	74.6	83.6
G	.075	66.7	74.7

From this table we see that the largest gradient occurs at point C, near the outside edge of the smaller radius of curvature, as one who is familiar with solutions of Laplace's equation would expect. The test was made, the disk was able to hold 90 KV DC before sparking occurred. From my previous calculations, 90 KV on the disk should result in 112x(.9)=100.8 KV/in at point C. At this point I swallow my pride and point out that value of electric field that causes sparking in air under atmospheric conditions is very close to 80 KV/in. My confidence in this technique ever so slightly shaken by this 20% discrepancy, I was most anxious to know at which point on the disk sparking did occur (as the next highest field when I measured resulted in only a 10% discrepancy). Thanks to an idea by J. Riedel and technical assistance by D. Johnson, the sparking test was reconducted, and this time a pinhole camera was used to record the spark arcs, to determine their origin. Lo and behold, the photographs (shown on a later page) reveal the fact that the sparking originated on the edge of the disk designated as the point C on my 2-dimensional model. This news was reassuring, as it agreed with my measurements as to where the largest gradient was. So I took heart at such qualitative agreement and set out to find the cause of the quantitative discrepancy.

So it seems that this resistive paper technique over estimates gradients by about 20%. This is most likely due to the fact that the conductive paint is not an especially good conductor, consequently the very edge of the paint doesn't represent the full voltage equipotential (100 KV or whatever), but the true 100% voltage curve lies a small distance in from the edge. Also, the finite thickness of the resistive paper and the fact that the conductive paint is only on one side is a significant factor in this error, since the current flow (and consequent IR voltage drop) in the paper doesn't stop exactly at the edge of the paint, but goes on a short ways, about 10 mils in this case. The question arises: is this difference between the apparent edge and the "effective edge" of the paint a constant factor or one that is dependent upon the value of the gradient that one is trying to measure? At first

I thought that the error involved here was proportional to the gradient measured so only a simple fudge factor was needed. But I had a model of what was going on that involved the solution of Laplaces equation subject to the finite thickness of the paper and the conductive paint equipotential on just one side of the paper. In this model, all lines of current flow (electric field lines) cannot terminate exactly on the edge of the paint, for it would produce too large a current density, but some current must flow past the edge and reach the paint from underneath. This would create an "effective edge" of the paint somewhat behind the apparent or visual edge of the paint. This implies (through the "Uniqueness theorem") that this difference between the apparent and "effective" edge of the paint is dependent only on the properties (thickness and conductivity) of the paper itself and is independent of the value of the gradient being measured. But which is "correct"?

After much head scratching (and cursing under my breath), a modest piece of detective work uncovered an obscure and previously misinterpreted bit of data. This was a measurement of the gradient between two parallel conductive lines on resistive paper, length one foot, separation of 2.94 inches. Since the length of the lines was over 4 times their separation, measurements between the lines should be essentially free of edge effects. The measured distance to both the 0.5 volt and 9.5 volt equipotentials from the ground and 10 volt lines, respectively, was 0.135". The expected value of the distance was (2.94)x(.05)=0.147 inches. In this case the difference between the effective and apparent edge of the paint was .012". In the sparking test model I measured at point C a distance to the 95% potential of .050", corresponding to a field of

100.8 KV/in with 90 KV on the disk, this value of the field already reflecting the so-called cylindrical correction, the nature of which I have laboriously elucidated. The actual value of the field was 80 KV/in, as it just caused sparking. So the distance to the 95% potential should be .063", since V=E/d or in numbers, (100.8)x (.050)$\cong$(80)x(.063). Notice that this difference between the measured distance and what should have been measured distance is .013", the difference between the effective and apparent edge of the paint. This is in very good agreement with the results of my parallel line measurement (of .012" difference) even though the parallel line measurement was for a field of only 34 KV/in. In this light I happily (if not hastily) conclude that this error of the edges is a constant of this particular paper, and that the extra .012 inches should be added to each of the distances I measured to find gradients, since I always measured the distances relative to the apparent edge of the paint. In this way I believe we may correctly interpret these resistive paper studies, and accurately predict the sparking behavior of the insulator corona ring, keeping in mind the important cylindrical correction factor that must be used when appropriate.

III. Possible Corona Ring Geometries

Now that I have expounded upon this resistive paper technique, I may present the results of the investigations made to determine the sparking voltage levels of different corona ring geometries. I shall organize these results in tables that give the fields at particular points of the various geometries assuming 100 KV on the corona ring. Illustrations appear in the back to show the different geometries and points used for field measurements.

All distances d to equipotentials already have been increased by .012" to correct for errors intrinsic to this resistive paper technique. I shall not burden the reader with the calculations for every cylindrical correction factor, the procedure has been explained already and is straight forward. It is enough to say that I use for the outer radius of the calculation the largest radius in the drawing that is the straight part of the outer conductor, not the tapered part. This will result in some overestimate of the gradients, since this enhancement factor increases with increasing separation of the inner and outer radii. I apply this correction only to the largest fields in each case, as these give us the limitations of each geometry. Finally, from these largest fields I calculate the largest voltage that each corona ring is able to hold without sparking, to give a comparison of all the cases.

The page titled "Original geometry" shows the outer conductor of the insulator cavity at R=10.88", but a test was made with R=11.88" also. So here are the results

Original Geometry, R_{out}=10.88"

Point	Distance to 95% of full V (in)	E, uncorrected (KV/in)	E corrected (KV/in)	V_{max} (KV)
A	.052	96.2	106.3	
B	.082	61.0	67.4	75.3
C	.062	80.7	89.2	

R_{out}=11.88

Point	d (in)	E, uncorrected (KV/in)	E corrected (KV/in)	V_{max} (KV)
A	.068	73.6	85.1	
B	.094	53.2	61.5	94.0
C	.077	65.0	75.2	

It appears that moving the outer conductor out an inch does not reduce the fields at the surface of the corona ring enought to allow it to hold 100 KV. So we try to remedy this by using larger radii of curvature on the shield, thereby giving the electric field lines terminating on the curve more room to "spread out", reducing the field strength at the surface. The drawing entitled "Modified Geometry I" shows two cases in one drawing, the case where R_1=1/2", R_2=11/16", and where R_1=5/8", R_2=3/4". The outer conductor is at R=11.88" in both cases.

Modified Geometry I

Point	d (in)	E, uncorrected (KV/in)	E, corrected (KV/in)	V_{max} (KV)
		R_1=1/2"	R_2=11/16"	
A	.099	50.5		
B	.072	69.4		
C	.068	73.5	84.7	94.4
D	.080	62.5		
E	.077	64.9		
		R_1=5/8"	R_2 = 3/4"	
A	.102	49.0		
B	.077	64.9		
C	.072	6[illegible].4	79.5	100.7
D	.075	66.7		
E	.077	64.9		

Increasing the radii of curvature did increase the capacity of the corona shield, so I made one more modification, increasing the radii of curvature to 1 1/8". This illustration is the "Modified Geometry II" page, and I tested this shape for two positions of the outer conductor, namely for R=10.88" and R=11.88".

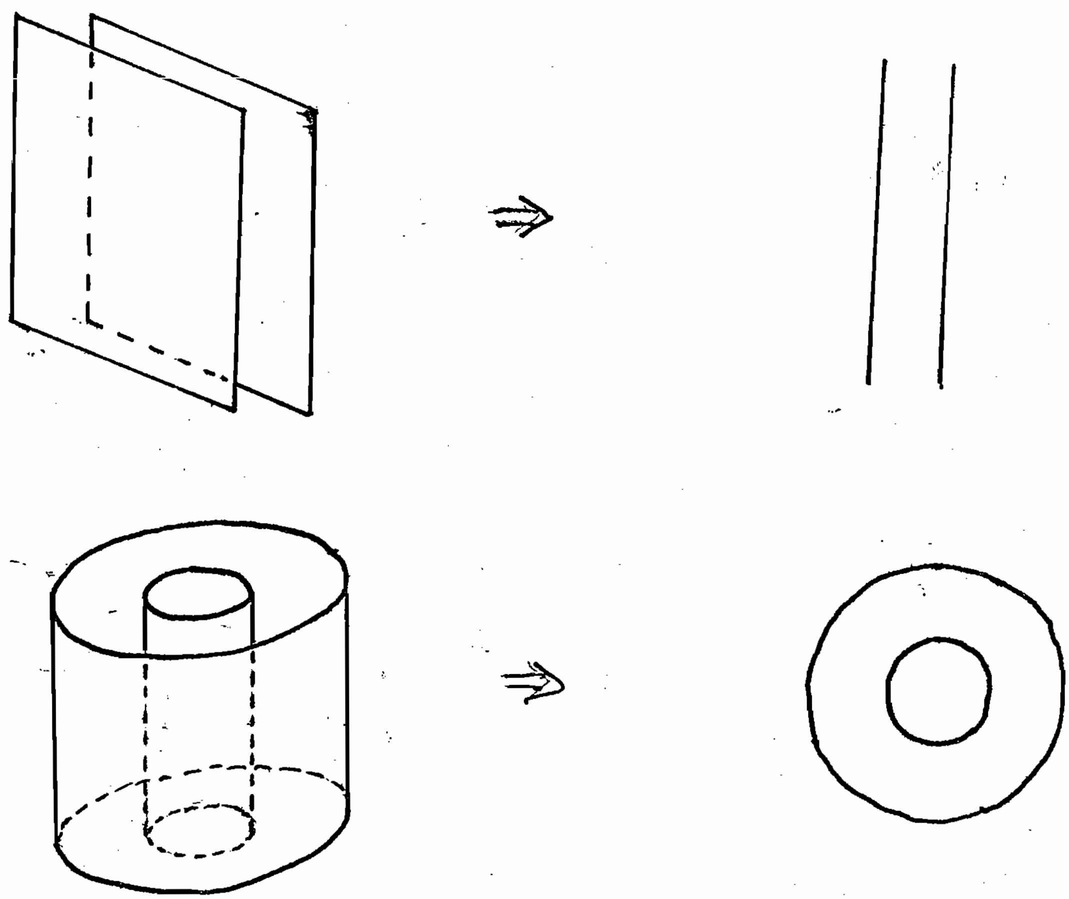
Modified Geometry II

Point	d (in)	E, uncorrected (KV/in)	E, corrected (KV/in)	V_{max} (KV)
		R_{out} = 10.88"		
A	.087	57.5		
B	.074	67.6		
C	.067	74.6		
D	.062	80.6		88
E	.059	84.7	91.0	
F	.062	80.6		
G	.067	74.6		
		R_{out} = 11.88"		
A	.107	46.7		
B	.084	59.5		
C	.087	57.5		
D	.080	62.5		
E	.077	64.9	73.0	109.6
F	.078	64.1		
G	.079	63.3		

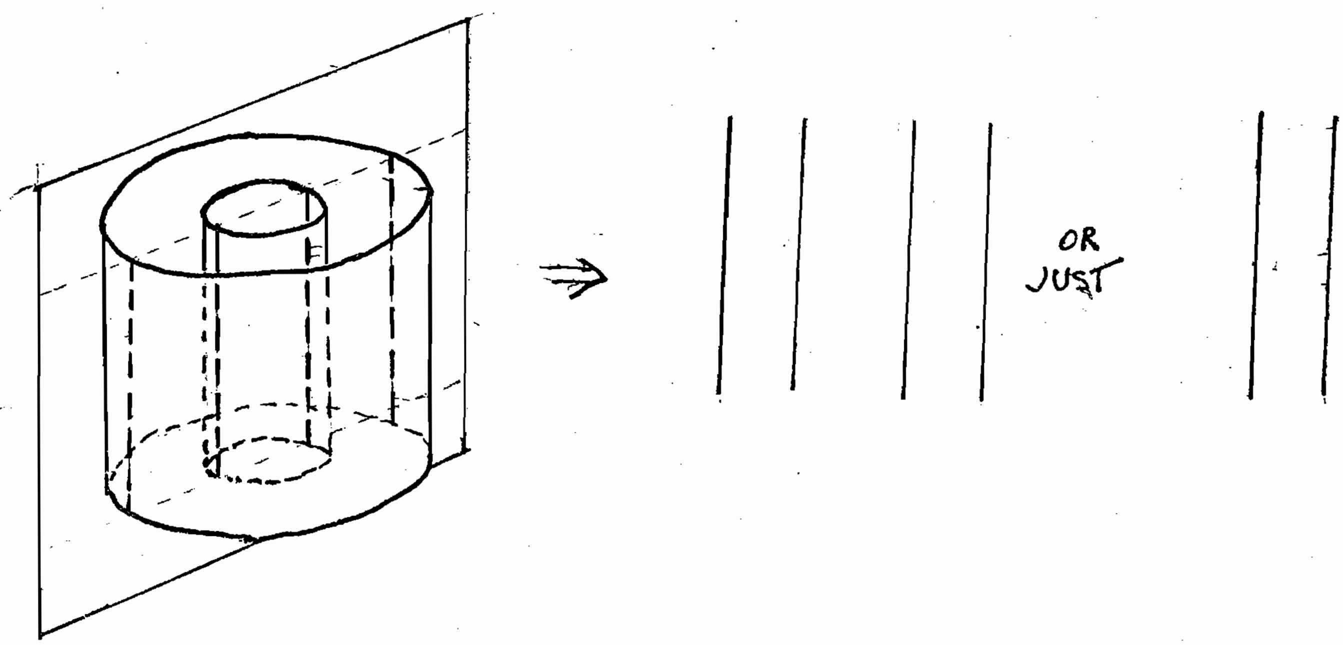
So we find that increasing the radii of curvature further gets us what we want, namely, an insulator corona ring that will hold over 100 KV. Unfortunately, it was necessary to increase the outer conductor radius to 11.88 inches (12 in O.D.). Since the machine is getting quite crowded near the insulator cavities, it would be nice to not have to make them any larger. It may be possible to make a polynomial interpolation to find some outer conductor radius that would enable the insulator shield to hold just 100 KV instead of 110 KV. Still, it would be nice to have some margin of safety in this sparking business.

Examples of accurate 2-dimensional representations

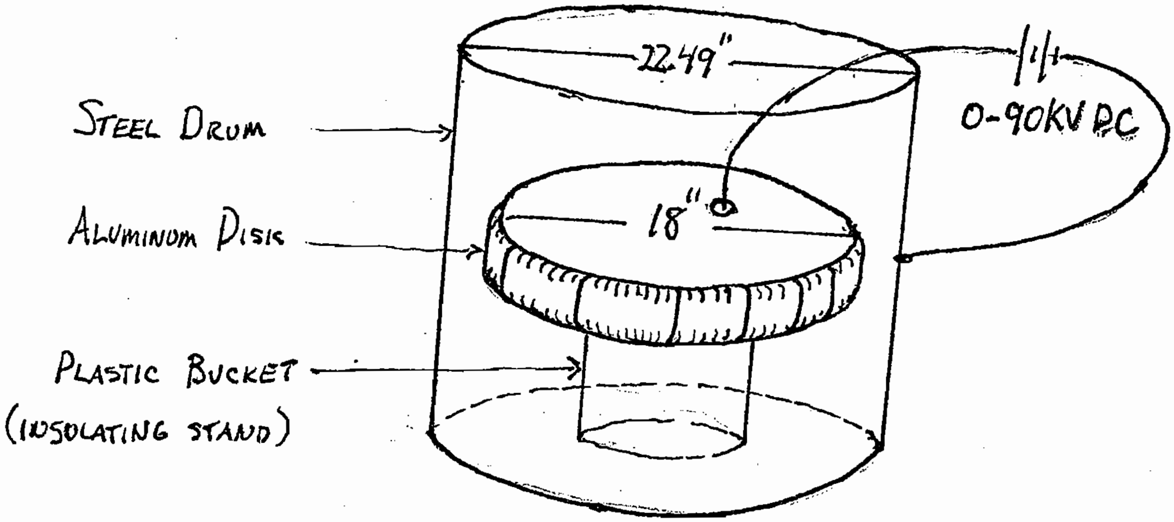
3-D 2-D



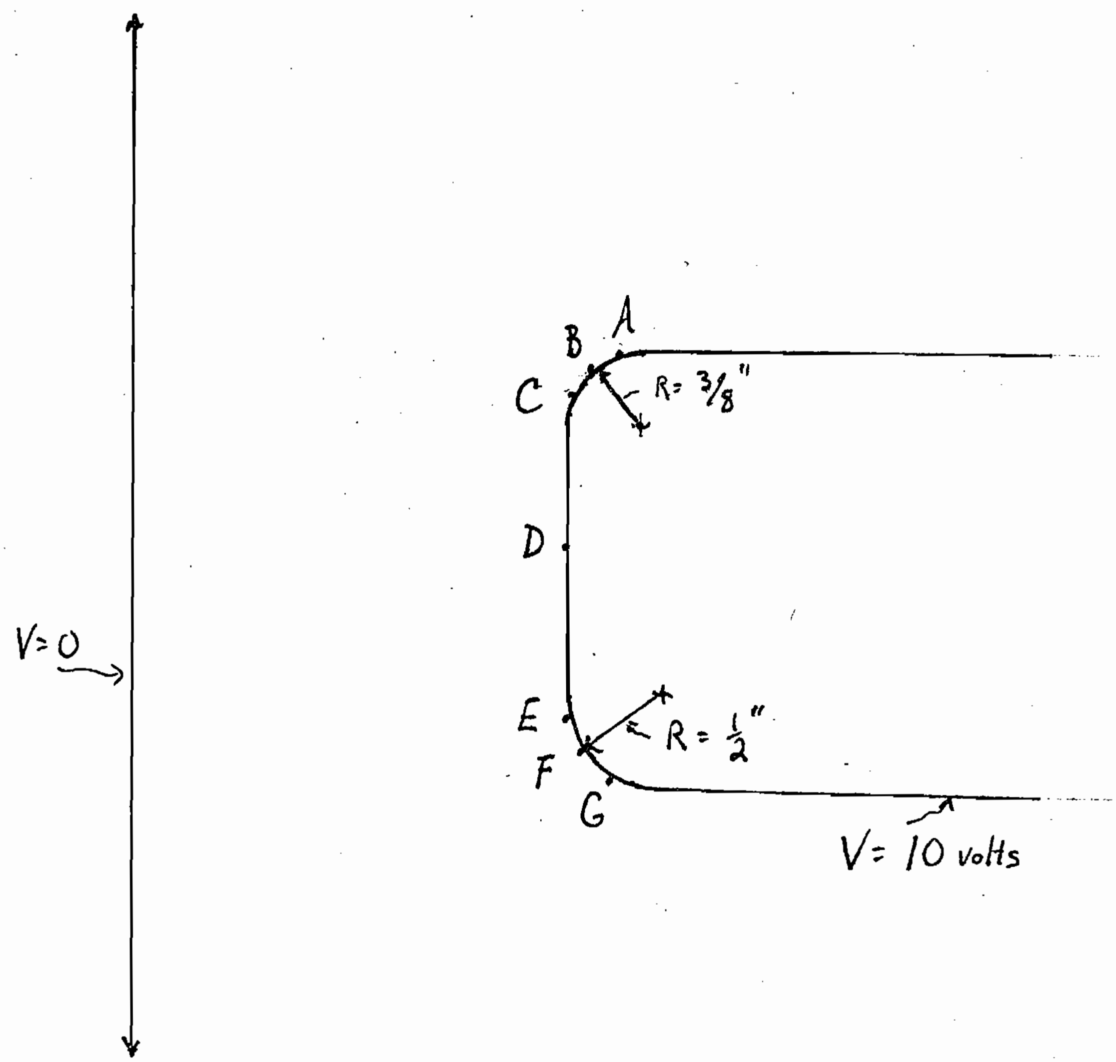
2-dimensional representation requiring cylindrical correction factor



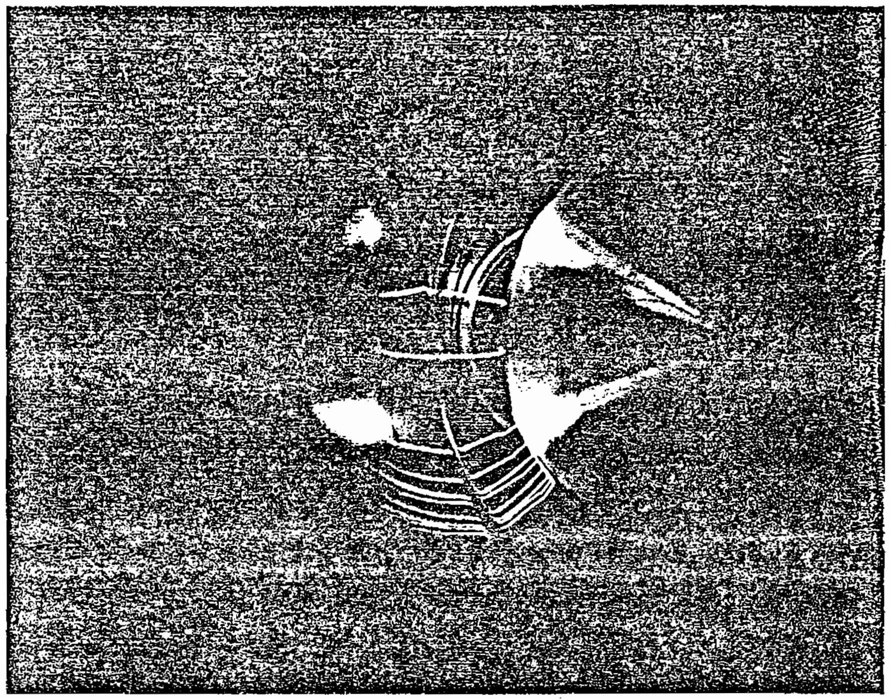
SPARKING TEST SETUP

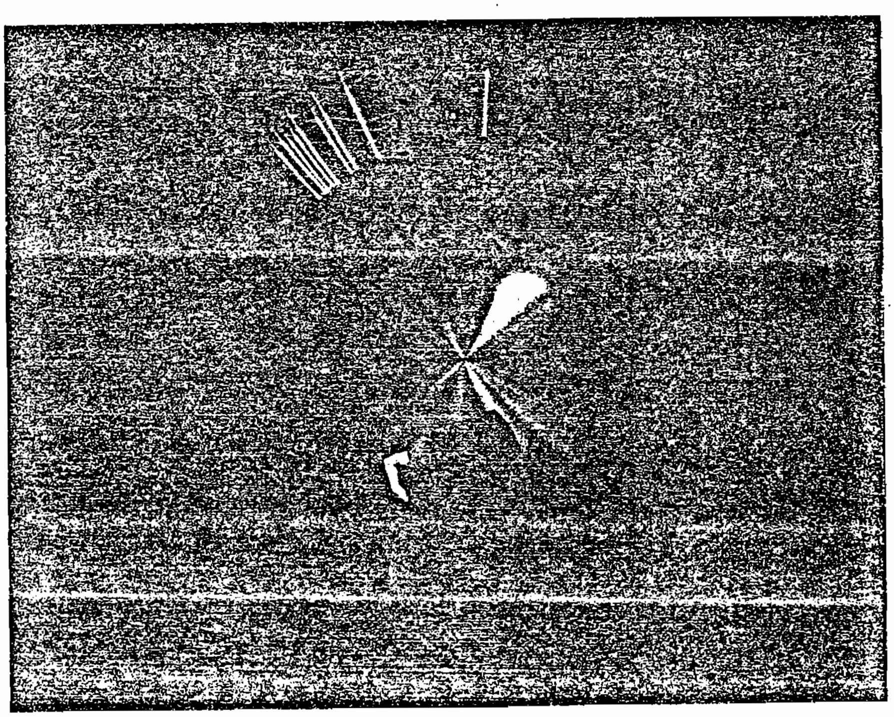


RESISTIVE PAPER MODEL OF TEST SET UP

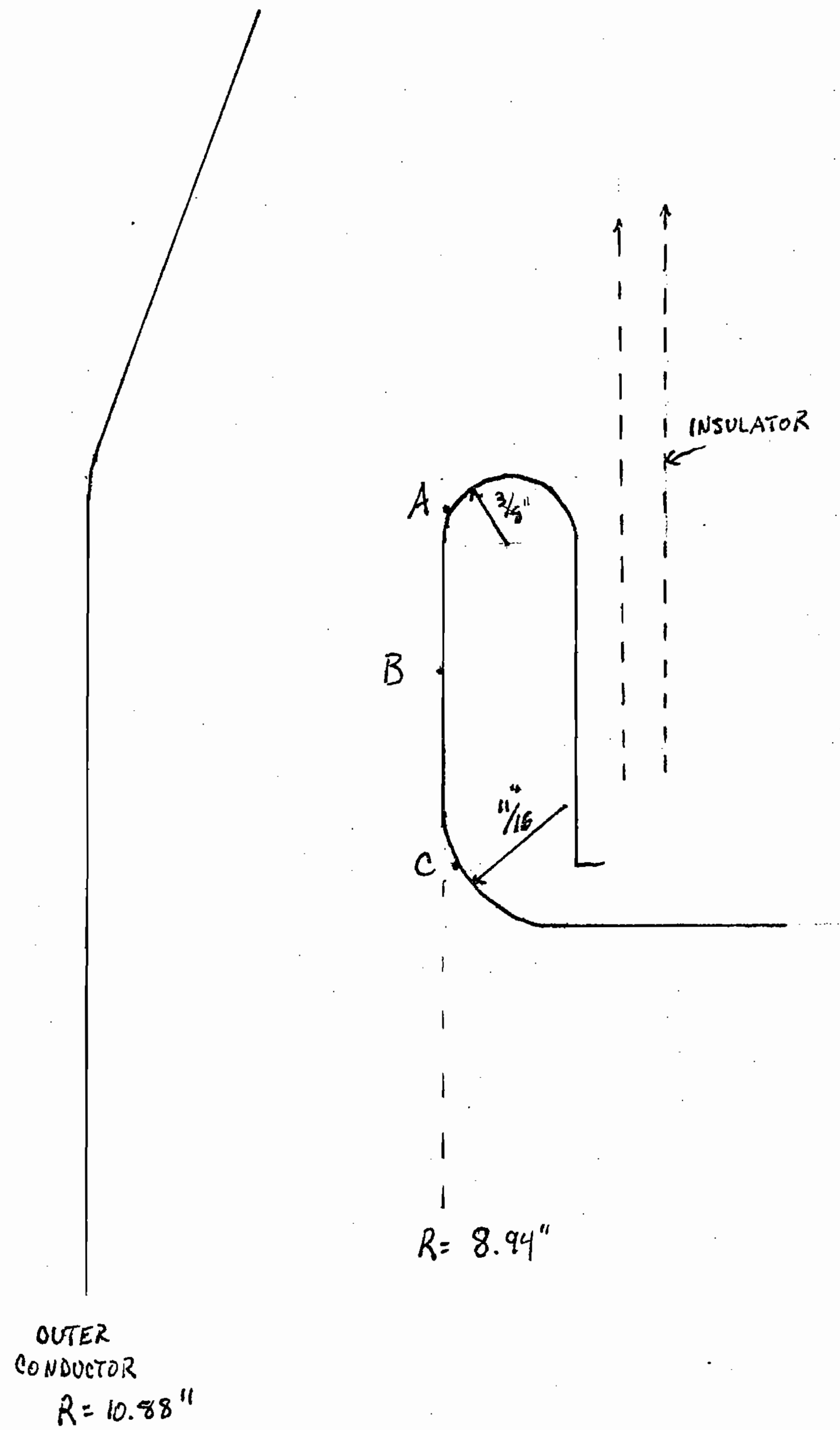


SPARKING TEST PHOTOGRAPHS
(COPIES ALREADY MADE)

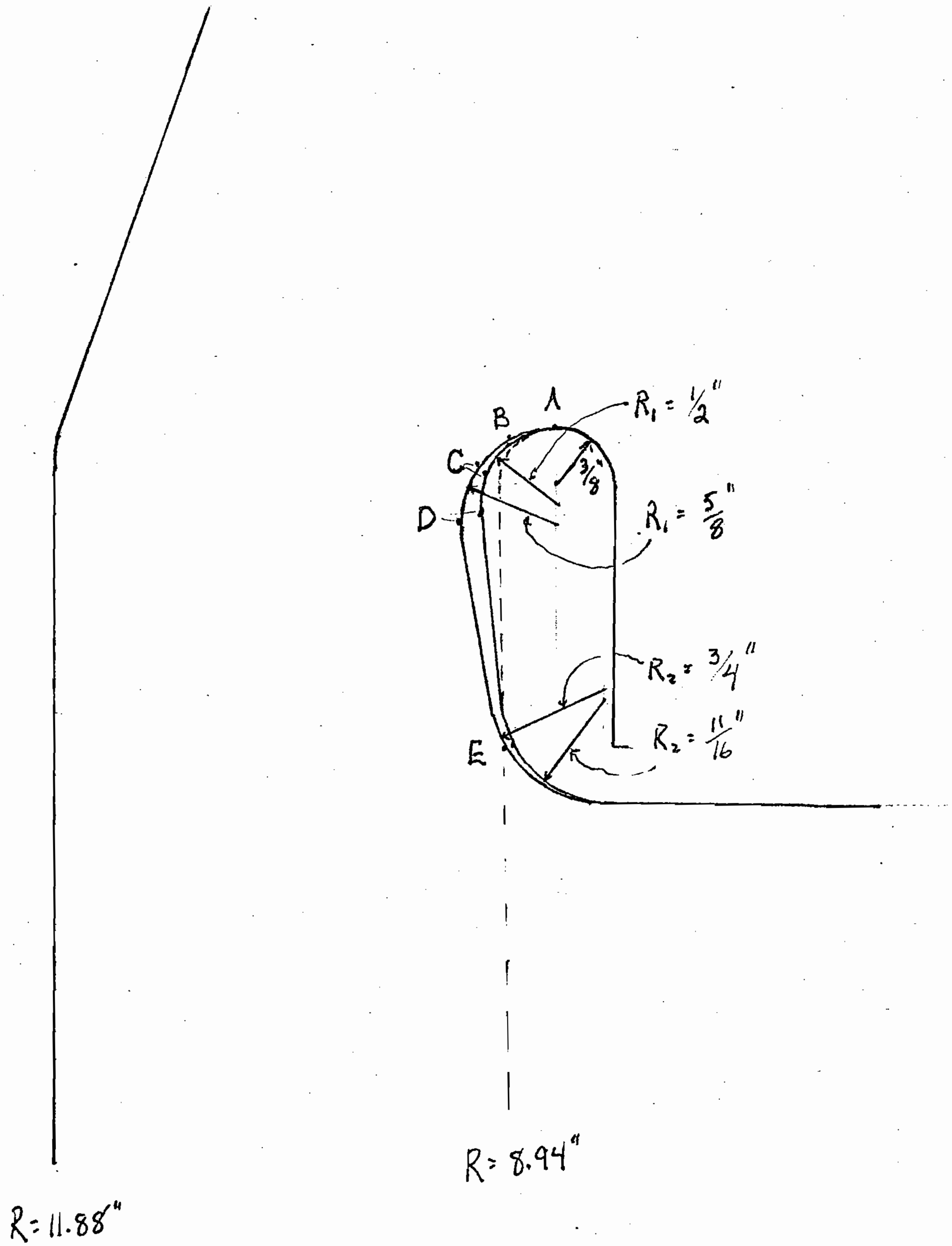




ORIGINAL CORONA RING GEOMETRY
(2-D REPRESENTATION)



MODIFIED CORONA RING GEOMETRY I



MODIFIED GEOMETRY II