214-0058-01-01_00

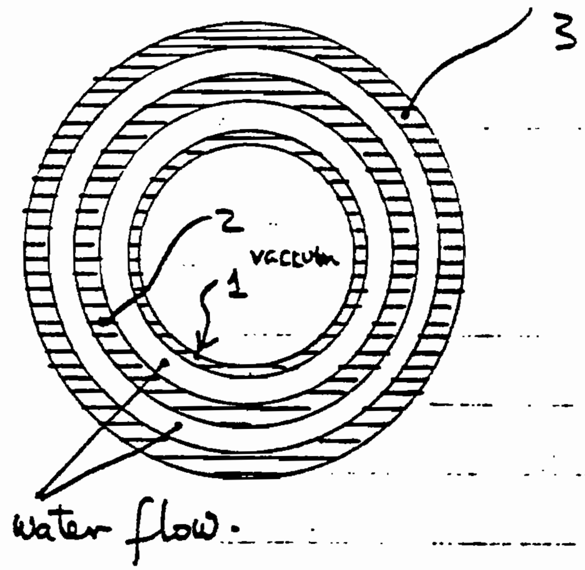
R.F. Book

R.F. Note #57

December 19, 1979
F. Marti

Felix Marti 12|19|79

Heat transfer in the cooling system of dee stem



Copper tubes

1) 3.125 O.D. (inches)
   2.945 I.D.

2) 3.625 O.D
   3.425 ID

3) 4.125 O.D
   3.857 I.D

17 ft long.

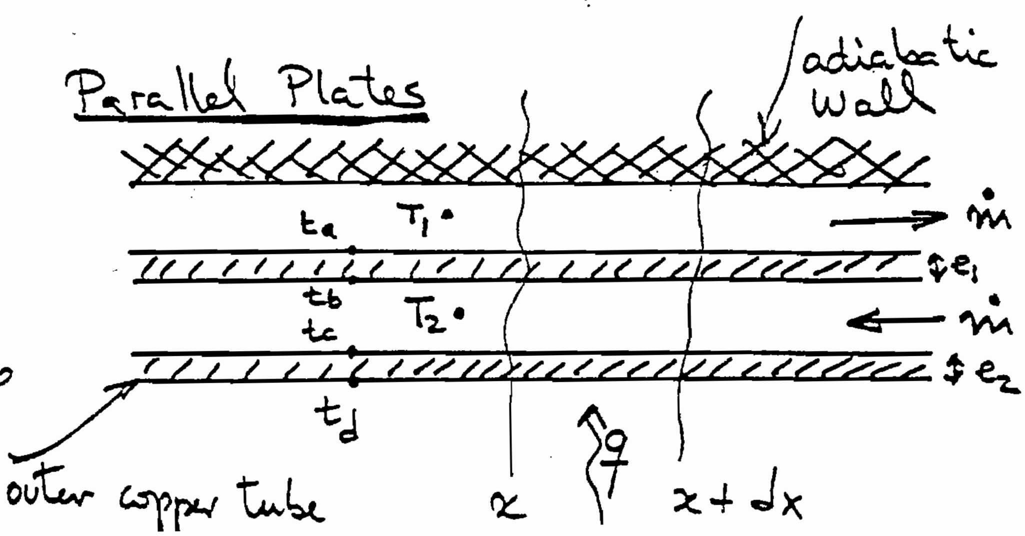
$Q \sim 4$ gallons/min ~ 15 liters/min ~ 15 kg/min

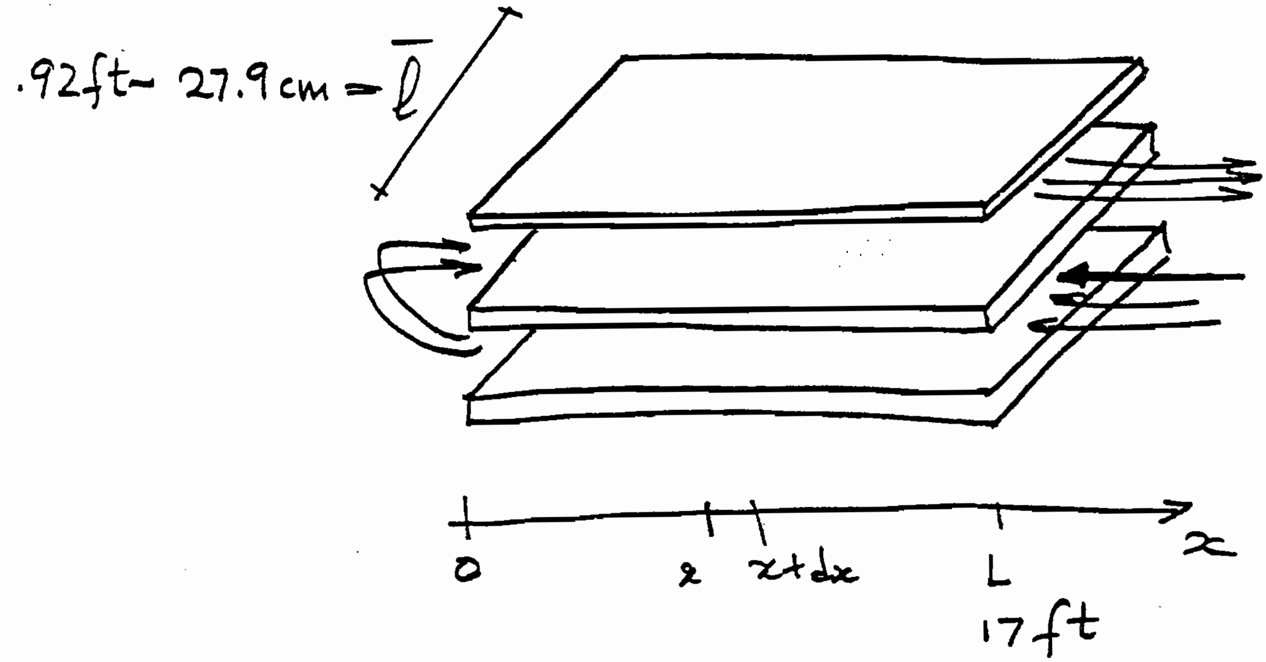
$$v_{water} \sim \frac{15 \times 1000\ cm^3/min}{9\ cm^2} \sim 1.67 \times 10^3\ cm/min \sim 28\ cm/s \sim 1\ ft/s$$

1st approximation. Parallel Plates

small t - copper temp

Cap. T - Water Temp





Power generated per unit length $= \dfrac{33 \text{ kW}}{17 \text{ ft}}$

Assume average water temperature $60°C = 140°F$

Water properties (from Holman)

$$c_p = 1 \ \frac{BTU}{lb_m \, °F}$$

$$\rho = 61.4 \ \frac{lb_m}{ft^3}$$

$$\mu = 1.14 \ \frac{lb_m}{ft\text{-}hr}$$

$$k_w = .378 \ \frac{BTU}{hr\text{-}ft\text{-}°F}$$

$$P_r = \frac{c_p \mu}{k} = 3.01$$

$$\text{Reynolds} = \frac{v\, d_e}{\nu} = \frac{v\, \rho\, d_e}{\mu}$$

$$d_e = \text{hydraulic radius} = \frac{4\ \text{flow section}}{\text{wetted perimeter}} \sim D' - D \sim .02\ \text{ft}$$

$$Re \sim \frac{1\ \text{ft/s} \times 61.4\ \text{lb}_m/\text{ft}^3 \times .02\ \text{ft}}{1.14\ \frac{\text{lb}_m}{\text{ft-hr}}\ \frac{1}{3600}\ \frac{\text{hr}}{\text{s}}} = 3880 \quad \text{turbulent flow}$$

<u>Equations</u> (Neglect transfer along the copper tube)

1) $$q\,dx = \frac{k\,dx\,\ell}{e_2}\,(t_d - t_c)$$

2) $$q\,dx = h\,dx\,\ell\,(t_c - T_2)$$

3) $$\dot{m}c_p\left[T_2(x) - T_2(x+dx)\right] + h\,(T_2 - t_b)\,dx\,\ell = h\,(t_c - T_2)\,dx\,\ell$$

4) $$h\,dx\,\ell\,(T_2 - t_b) = k\,\frac{dx\,\ell}{e_1}\,(t_b - t_a)$$

5) $$k\,\frac{dx\,\ell}{e_1}\,(t_b - t_a) = \dot{m}c_p\left[T_1(x+dx) - T_1(x)\right]$$

6) $$\dot{m}c_p\left[T_1(x+dx) - T_1(x)\right] = h\,(t_a - T_1)\,dx\,\ell$$

from which we get

$$t_a = T_1 + \frac{\dot{m} c_p}{h\ell} \frac{dT_1}{dx}$$

$$t_b = T_1 + \frac{\dot{m} c_p}{h\ell}\left(1 + \frac{e_1 h}{k}\right)\frac{dT_1}{dx}$$

$$t_c = T_2 + \frac{q}{h\ell}$$

$$t_d = t_c + \frac{q e_2}{k\ell}$$

$$T_2 = T_1 + \frac{\dot{m} c_p}{h\ell}\left(2 + \frac{e_1 h}{k}\right)\frac{dT_1}{dx}$$

$$\frac{d^2 T_1}{dx^2} = -\frac{q\,\ell h}{(\dot{m} c_p)^2\left(2 + \frac{e_1 h}{k}\right)} = -\alpha$$

Integrating

$$T_1(x) = -\frac{\alpha x^2}{2} + \beta x + \gamma$$

Boundary conditions: $T_1(0) = T_2(0)$

$T_2(L) = T_0$ = entrance temperature.

$$T_2(x) = T_1(x) + \frac{q}{\alpha \dot{m} c_p}\frac{dT_1}{dx} \qquad \frac{dT_1}{dx} = -\alpha x + \beta$$

$$T_1(0) = T_2(0) \Rightarrow \frac{dT_1}{dx}(0) = 0 \Rightarrow \beta = 0.$$

$$T_0 = -\frac{\alpha L^2}{2} + \gamma + \frac{q}{\alpha \dot{m} C_p}\left(-\alpha L\right)$$

$$\gamma = T_{max} = T_0 + \frac{\alpha L^2}{2} + \frac{qL}{\dot{m} C_p}$$

Numerical values:

$$q = \frac{33\ kW}{17\ ft} = \frac{33 \times 10^3}{5.168\ m} = 6.39 \times 10^3\ W/m$$

$$\dot{m} = \frac{15}{60} = .25\ kg/s = .55\ \frac{lb_m}{s}$$

$$C_p = 1\ \frac{BTU}{lb_m\ °F} =$$

$$1\ \frac{BTU}{hr\ ft\ °F} = \frac{1}{57.8}\ \frac{W}{cm\ °C}$$

$$1\ \frac{BTU}{°F} = \frac{30.4 \times 3600}{57.8}\ \frac{W\,s}{°C} = 1893\ \frac{W.s}{°C}$$

$$\Rightarrow C_p = 1893\ \frac{W.s}{lb_m\ °C}$$

$$\frac{qL}{\dot{m} C_p} = \frac{6.39 \times 10^3\ W/m \times 5.17\ m}{.55\ \frac{lb_m}{s}\ \ 1893\ \frac{W.s}{lb_m\ °C}} = 32\ °C$$

To find h we use the relation:

$$\frac{h\,d_e}{k_w} = .023 \left(\frac{d_e v}{\nu}\right)^{.8} \left(\frac{c_p \mu}{k_w}\right)^{1/3}$$

$$h = .023 \times \frac{.378\ BTU/hr\text{-}ft\text{-}°F}{.02\ ft} \left(3880\right)^{.8} \left(3.01\right)^{1/3}$$

$$h = 466 \frac{BTU}{hr\text{-}ft^2\text{-}°F}$$

For the copper $\quad k_{copper} \sim 220 \dfrac{BTU}{hr\text{-}ft\text{-}°F}$

$$\frac{e_1 h}{k} = \frac{.0083 \times 466}{220} = .018$$

$$\alpha = \frac{q\,\ell\,h}{(\dot{m} c_p)^2 \left(2 + \frac{e_1 h}{k}\right)} =$$

$$= \left(\frac{qL}{\dot{m} c_p}\right) \frac{\ell}{L} \frac{h}{\dot{m} c_p \times 2.02} = 32°C\ \frac{.92}{17}\ \frac{466 \frac{BTU}{hr\text{-}ft^2\text{-}°F}}{.55 \frac{lb_m}{s}\ 1.\frac{BTU}{lb_m °F}\ 2.02}$$

$$= .20\ °C/ft^2$$

$$T_{max} = T_0 + \frac{\alpha L^2}{2} + \frac{qL}{\dot{m} c_p} =$$

$$= 20° + \frac{.2 \times (17)^2}{2} + 32$$

$$= 20 + 29 + 32 = 81\ °C$$

$$\boxed{T_1 = 81 - \frac{x^2}{10}\ °C} \quad (x \text{ in ft})$$

$$T_2 = T_1 - \frac{q}{\dot{m} c_p} x = 81 - \frac{x^2}{10} - 1.88\, x$$

$$\boxed{T_2 = 81 - \frac{x^2}{10} - 1.88\, x \quad °C} \quad (x \text{ in ft})$$

$$t_c = T_2 + \frac{q}{h\ell} \qquad \frac{q}{h\ell} = \frac{6.39 \times 10^3\ W/m \quad \frac{1}{100} \frac{m}{cm}}{466\ \frac{BTU}{hr\ ft^2\text{-}°F} \times .92\ ft \quad \frac{1\ W/cm\cdot°C}{57.8\ \frac{BTU}{hr\ ft\ °F}}}$$

$$\frac{q}{h\ell} = 8.6\ °C$$

$$\boxed{t_c = T_2 + 9\ °C}$$

$$t_d = t_c + \frac{q\ell_2}{k\ell} \approx t_c$$

We can verify if the assumption that there was no transfer along the copper tube is reasonable.

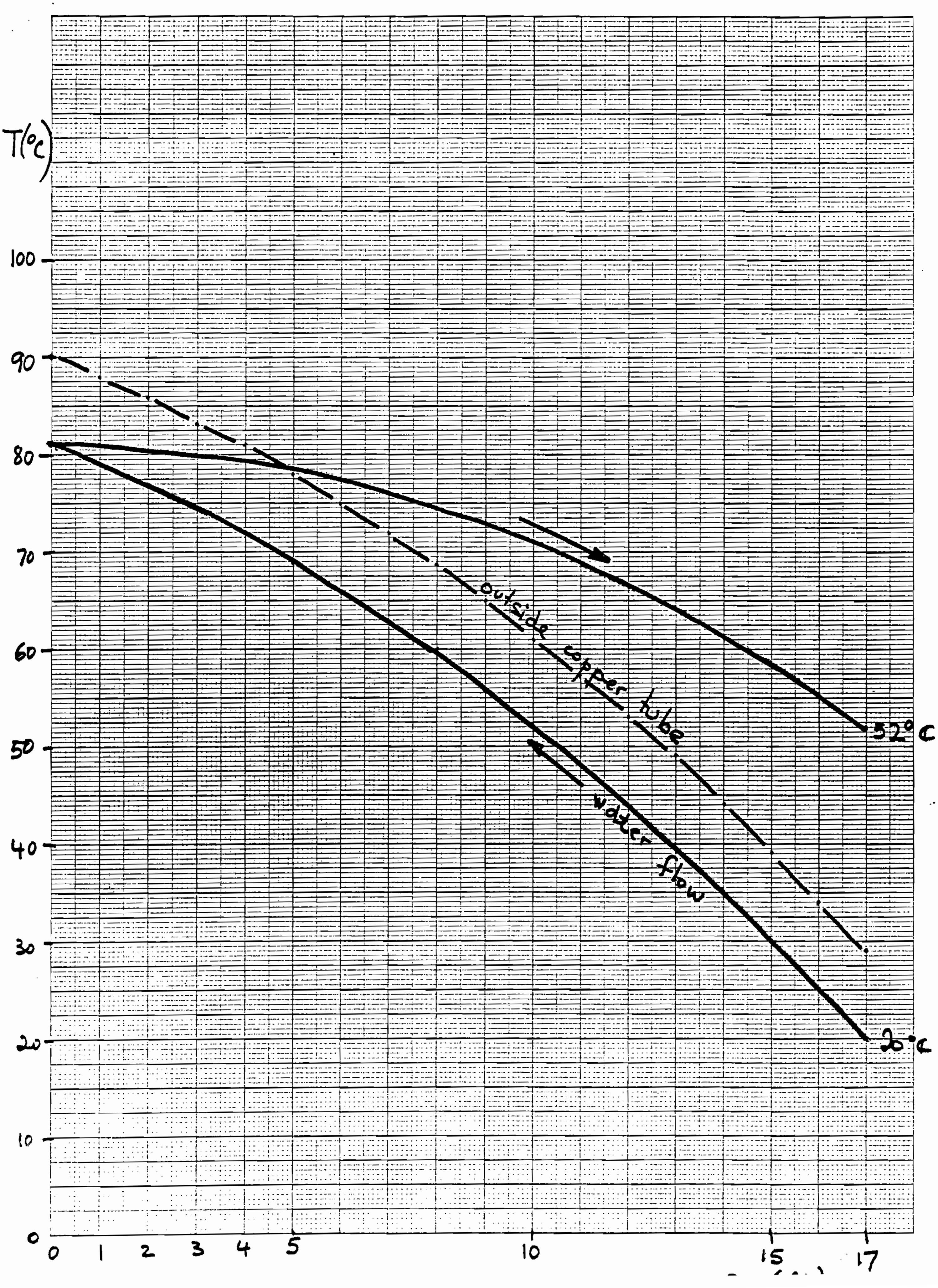
$$\Delta T = 90 - 29 = 61^\circ C$$

$$\text{Area} \cong 11 \text{ cm}^2 \qquad k = 220 \ \frac{BTU}{hr\ ft\ ^\circ F}$$

$$L = 17 \text{ ft}$$

$$Q_{copper} = k A' \frac{\Delta T}{L} = \frac{220}{57.8} \frac{W}{cm\ ^\circ C} \times \frac{11 \text{ cm}^2 \times 61\ ^\circ C}{17 \times 30.4 \text{ cm}}$$

$$= 5 \text{ Watts.}$$ negligible



T(°C)
100
90
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
10
15
17
outside copper tube
water flow
52°C
20°C

Let's assume now that the average temperature is higher.

$t_{average\ water} = 70°C \sim 160°F$

$$c_p = 1 \quad \frac{BTU}{lbm\ °F}$$

$$\rho = 61. \quad \frac{lbm}{ft^3}$$

$$\mu = .97 \quad \frac{lbm}{ft\ hr} \qquad k_w = .38$$

$$Pr = 2.53$$

$$Re = \frac{v\,d\,\rho}{\mu} = \frac{1\ ft/s \times 61 \times .02 \times 3600}{.97} = 4528$$

$$\frac{qL}{\dot{m}\,c_p} = 32°C \text{ same}$$

$$h = \frac{.023 \times .38}{.02} (4528)^{.8} (2.53)^{1/3} = 500 \quad \frac{BTU}{hr - ft^2 - °F}$$

$$\alpha = .2 \times \frac{500}{466} = .215$$

$$T_{max} = 20 + \frac{.215\ (17)^2}{2} + 32 = 83°C$$

10° change in average temperature gives a 2°C change in the maximum temperature