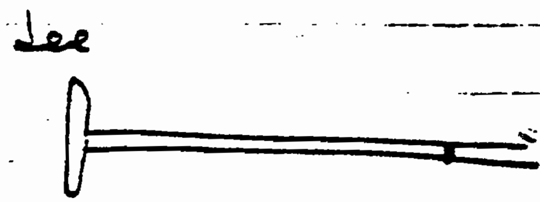
RF Book

Felix Marti 1/22/80 (1)

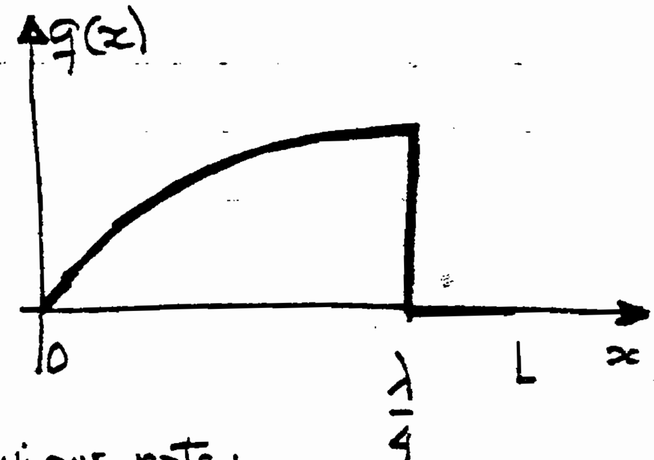
Second Approximation to the problem of. Heat transfer in the dee stem.

We had previously assumed that the power dissipated per unit length on the inner cylinder of the dee stem was constant

We will consider now a more realistic distribution



$$q(x) = \begin{cases} Q \sin^2 \frac{2\pi x}{\lambda} & 0 < x < \frac{\lambda}{4} \\ 0 & \frac{\lambda}{4} < x < L \end{cases}$$



The $x = \frac{\lambda}{4}$ point corresponds to the sliding short.

Using the equations found in our previous note:

$$\frac{d^2 T_1}{dx^2} = -\alpha(x) = -\frac{\ell h}{(\dot{m} c_p)^2 \left(2 + \frac{e_1 h}{k}\right)} q(x)$$

$$\frac{d^2 T_1}{dx^2} = -\tau \sin^2\left(\frac{2\pi x}{\lambda}\right) \quad \text{with} \quad \tau = \frac{\ell h Q}{(\dot{m} c_p)^2 \left(2 + \frac{e_1 h}{k}\right)}$$

integrating:

$$T_1(x) = -\tau\left[\frac{x^2}{4} + \frac{\lambda^2}{32\pi^2}\cos\left(\frac{4\pi x}{\lambda}\right)\right] + \beta' x + \gamma'$$

$$T_2(x) = T_1(x) + \frac{\dot{m}c_p}{\ell h}\left(2 + \frac{e_1 h}{k}\right)\frac{dT_1}{dx}$$

Boundary conditions: $T_2(0) = T_1(0) \Rightarrow \frac{dT_1}{dx}(0) = \beta' = 0$

For $\frac{\lambda}{4} < x < L$ $\quad q = 0 \quad \frac{d^2T_1}{dx^2} = 0$

$$T_1 = c_1 x + c_2$$

$$T_2 = c_1 x + c_2 + \frac{\dot{m}c_p}{\ell h}\left(2 + \frac{e_1 h}{k}\right)c_1$$

$T_2(L) = T_0 =$ temperature entrance water. $\approx 20°$.

Solving for γ'

$$\gamma' = -\tau\frac{\lambda^2}{32} + T_0 + \frac{Q\lambda/8}{\dot{m}c_p} + \tau\frac{\lambda}{8}L + \frac{\tau\lambda^2}{32}\left(\frac{1}{2} - \frac{1}{\pi^2}\right)$$

$$T_{max} = \gamma' - \frac{\tau\lambda^2}{32\pi^2} = T_0 + \frac{Q\lambda/8}{\dot{m}c_p} + \tau\frac{\lambda}{8}L - \frac{\tau\lambda^2}{32}\left(\frac{1}{2} + \frac{2}{\pi^2}\right)$$

The total power dissipated is

$$W = \int_0^{\lambda/4} Q \sin^2\left(\frac{2\pi x}{\lambda}\right) dx = Q\left(\frac{\lambda}{8}\right)$$

Case (a) $\frac{\lambda}{4} = 17$ ft. (low frequency) $W = 33$ kW

Using the values found in the 1st approximation study we obtain

$$T_{max} = T_0 + 50 \quad °C$$

$$T_1(x) = 70 - \frac{x^2}{10} + 5.9\left[1 - \cos\left(\frac{x}{5.41}\right)\right]$$

Fig 1 shows the water temperature as a function of x.

Case (b) $\frac{\lambda}{4} = 5$ ft (high frequency) $W = 33$ kW

$$T_{max} = T_0 + 78 \quad °C$$

$$T_1(x) = 98 - .34x^2 + 1.73\left(1 - \cos\frac{x}{1.6}\right)$$

for $0 \leq x < \frac{\lambda}{4}$ and changes linearly with x for $\frac{\lambda}{4} < x < L$. Fig 2 shows T vs. x.

After we made these graphs new information indicated that the maximum power was 20 kW and not 33 as we had assumed, and that the water flow was close to 3.4 gallons per minute and not 4.

Taking into account possible changes in these parameters the maximum temperature can be found to be given by:

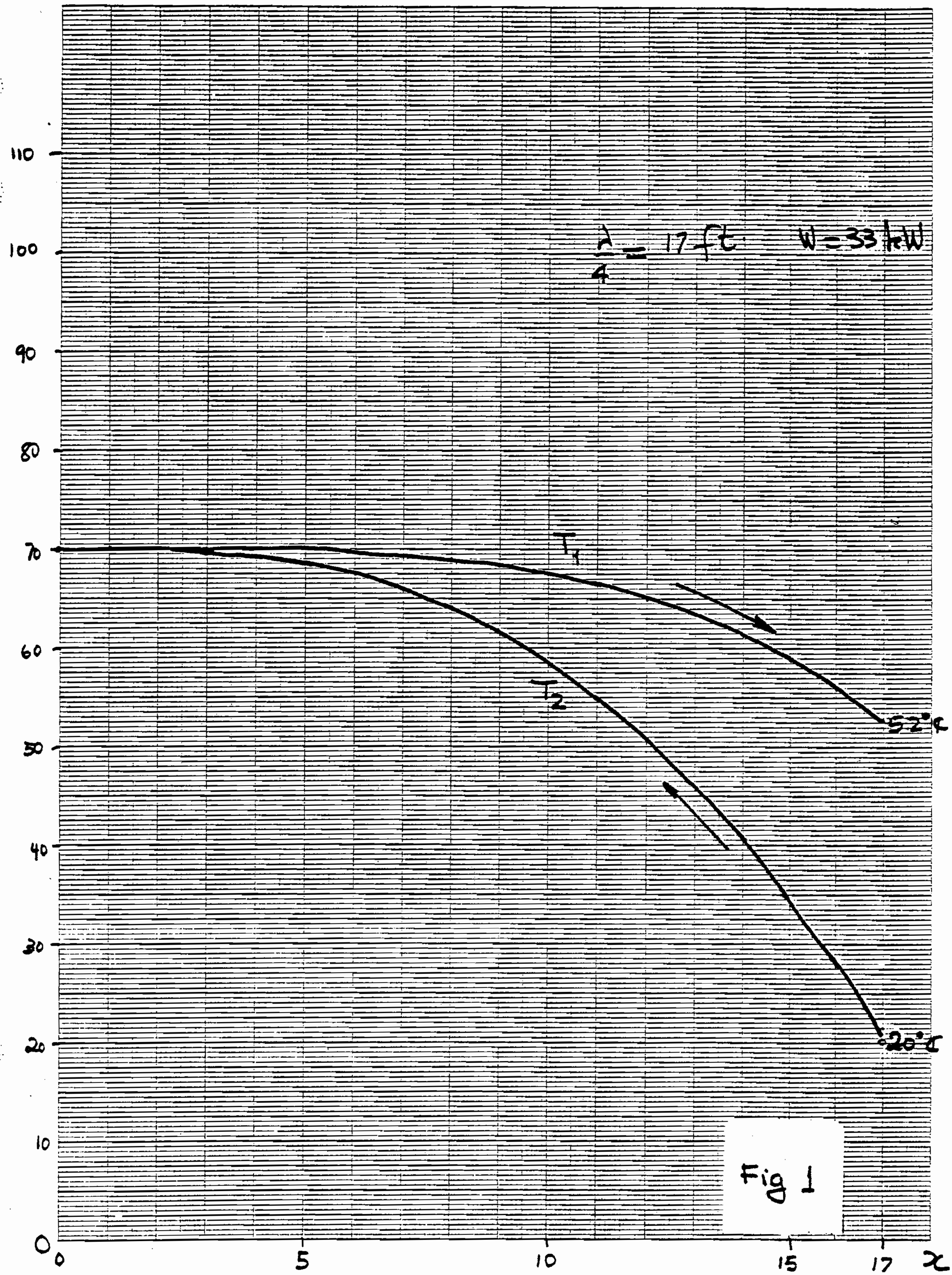
$$T_{max} = T_0 + 3.9\,\frac{W}{g} + 22.3\,\frac{W}{g^2}$$

Where W = total power dissipated on inner conductor of dee stem.

g = gallons per minute of water.

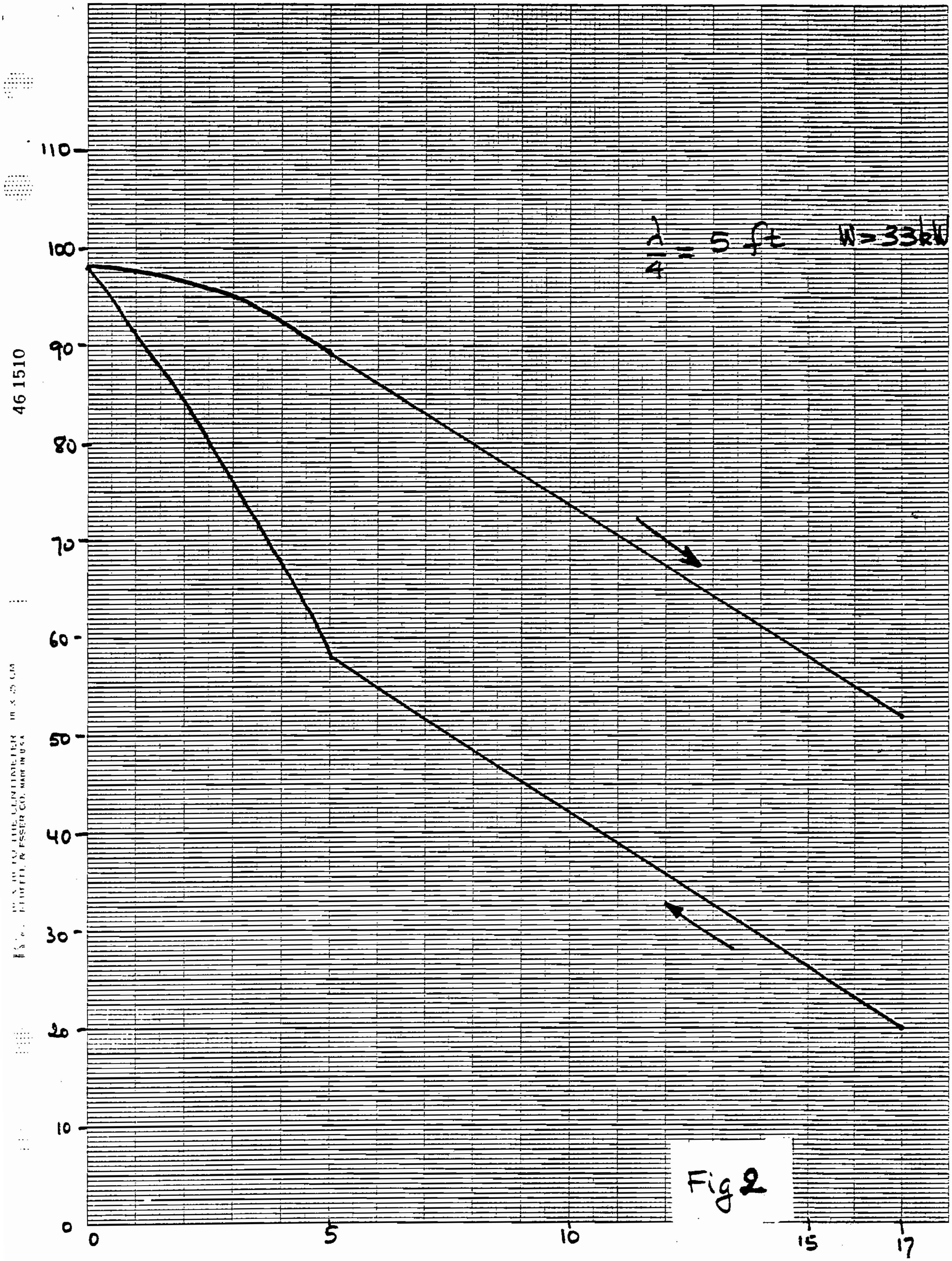
The worst case occurs for the high frequency case. For W = 20 kW and g = 3.4 gpm $T_{max} = T_0 + 62\,°C$ with $T_0 = 20\,°C$

$$T_{max} \cong 82\ °C$$



$\frac{\lambda}{4} = 17$ ft
W = 33 kW
T_1
T_2
52°C
20°C
0
10
20
30
40
50
60
70
80
90
100
110
0
5
10
15
17
x

Fig 1



$\frac{\lambda}{4} = 5$ ft
W > 33kW
110
100
90
80
70
60
50
40
30
20
10
0
0
5
10
15
17
46 1510

Fig 2