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RF Note 114
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# Notes on Shielded Coil Design

## Introduction

Past experience with large helical inductors, such as the K1200 B+ filter choke coils, has led to some methods for approximating the lumped equivalent circuit of these elements. Initial efforts involved trying to estimate the self-resonant modes of the coils, as well as the equivalent shunt circuit model. This note lists some approximations useful for design, and gives an example of coil analysis using some generic design rules in conjunction with NSCL software.

## Coil Resonance

For the coils discussed in this note, the physical dimensions are on the same order as a quarter wavelength of the operating frequency. This means we have to be concerned with self-resonant modes. It is instructive to try and represent the resonant behavior by two different processes.

The first representation is simply to assume that the coil, with its shield, forms a transmission line with a helical center conductor. Tables are available to estimate the lumped circuit equivalent model, which leads to a value for the quarter-wave resonance. The system will, of course, also resonate at quarter-wave multiples like any other shorted transmission line. Equivalent circuit estimates can also be made using a combination of software and empirical rules and an example of this method is given later in the note. So, for a simple shielded coil, shorted on one end and open on the other, this method will tell you where resonance occurs.

If, however, the coil is shorted on one end, open on the other and is essentially unshielded, resonance phenomena will still exist. One way to describe this is to assume that adjacent pairs of windings along the helix act as a transmission line pair. Each pair, though is also coupled to the next, and so on. Trying to analyze the system like this would be complicated, so a good rule of thumb was empirically tested instead. If you consider only one set of adjacent windings as a transmission line pair, the effective line length is half the linear (unwound) length of the coil. This implies that the lowest order self-resonant mode is estimated by taking one half the total linear length of the coil and equating this to $\lambda/4$. In other words,

$$f_0 = \frac{c}{2l}$$

where $l$ = linear length of the helix

The interesting thing about an unshielded large coil is that is will also possess a $\lambda/2$ like mode as well. This does not seem intuitive since we are assuming one end is shorted, while the other is left open. These boundary conditions would seem to force only odd multiples of a quarter-wave mode. The voltage and current distributions of this 'effective' $\lambda/2$ mode actually look like those of a half-wave line terminated with a capacitance instead of a short. This makes sense when we assume the adjacent conductor pairs model of the t-line. This type of mode can result in a voltage standing wave with a maximum at the coil center.

## Circuit Model Example 1

This section shows an example of a shielded coil analysis using some empirical relations, software, and a bit of hand waving to determine the equivalent shunt circuit. These results are then compared with measured data.

The shielded coil has the following dimensions:

| | |
|---|---|
| Axial Coil Length | 6" |
| Turns | 14 |
| Coil Tubing O.D. | 0.25" |
| Coil O.D | 1.25" |
| Shield O.D. | 3" |

On end is shorted and the other is left open by connecting it to a BNC connector, which possesses several pF shunt capacitance. The analysis of the coil is listed below with accompanying discussion.

Unshielded Coil Inductance (COIL program) L=1.08uH

Unshielded Coil R-factor (COIL) Rf=2.32

Shield Reduction Factor Sf=0.7

This shield reduction factor is empirical and only applies to similar geometries. There are tables available for estimating this effect for coaxial shield/coil pairs.

Adj. Total Coil L $L_T = 1.08(0.7) = 0.76uH$

Capacitance Estimate:
Assume the coil is a solid center conductor of 1.25" O.D. running down the middle of the shield. Using T-line equations,

$$Z_0 = 60\ln(\frac{D}{d}) = 52.53\Omega$$

Distributed Capacitance $C'_c = \frac{1}{Z_0 c} = 63.5pF/m$

Total Coil Capacitance $C_T = C'_c(6" \times .0254) = 9.7pF$

Distributed Inductance $L' = \frac{0.76uH}{6" \times .00254} = 4.99uH/m$

Equivalent Characteristic Impedance $Z_0 = \sqrt{\frac{L'}{C'}} = 280\Omega$

The appropriate RESON model for this helical line is created with the rectangular transmission line model (TR). The characteristic impedance of this line is the equivalent $Z_0$ listed above, with a t-line length equal to the axial coil length. Because of the helical center conductor, however, the RF length of the line is longer than the axial length of the coil. Similarly we can say the speed of the transverse wave is slower. To create the appearance of reduced transverse velocity, we can alter the dielectric constant in the RESON rectangular t-line. The velocity is proportional to the square root of the dielectric constant which gives the following relation.

$$\sqrt{\varepsilon r} = \frac{l_{coil}}{l_{axial}} = \frac{14(1.25")\pi}{6"} \Rightarrow \varepsilon r = 84$$

The next important parameter to determine is the circumference of each conductor which determines the losses in the system. These are the variables Wa and Wc in RESON. Following the normal method for these calculations, the outer conductor equivalent circumference is equal to the true circumference, reduced by the R-factor. The R-factor in this case is the number determined for the *coil* in free space, not an R-factor determined for the outer conductor, which we can't compute separately. We are assuming here that a bunching of the surface currents on the inner conductor coil will be mirrored on the surface of the outer conductor because the 'bunched' electric fields will distribute the surface currents in this manner on both conductors. The effects, of course, would be diluted as the shield is positioned further from the helix, so this is an approximation for a geometry where the distance between the coil and shield is of the same order as the helix diameter.

An additional reduction in the circumference has to be added to account for the fact that the t-line length we are inputting in RESON is the axial length. Losses, however, occur along the entire coil length. We have to introduce a factor which reduces the circumference (increases losses) by a ratio of the coil length to the axial length. This ratio has already been computed above and is equal to the square root of the equivalent dielectric constant. The entire relation for the outer conductor circumference (width) is written below.

$$Wc = \frac{\pi D}{R_f\sqrt{\varepsilon_r}} = \frac{3\pi}{R_f\sqrt{\varepsilon_r}} = 0.443"$$

The inner conductor circumference is computed in a similar manner, except with one additional factor. Because the R-factor was determined for a coil in free space, we multiply this factor by two (increase the losses) because the presence of the shield will increase the bunching on the surface of the coil which faces the outer conductor. Again, this is a ballpark number, which may only be reasonable for geometries similar to the one in this example.

$$Wa = \frac{\pi d}{2R_f\sqrt{\varepsilon_r}} = \frac{0.25\pi}{R_f\sqrt{\varepsilon_r}} = 0.018"$$

The RESON rectangular t-line input line would look like this (for a connection between nodes 1 and 2 with ground at node 0):

TR1 1 2 0 Wa=0.018 Wc=0.443 L=6 Z0=280 E=84

The results of the RESON simulation were compared with measured results of the actual device. The measurements were performed by attaching the 'open' end of the coil to a BNC connector and measuring the complex impedance at this terminal with the vector impedance meter. The results of these measurements were then used to determine Q and the equivalent shunt circuit. The comparison is shown in the table, with the p subscript on the various elements indicating the element in a parallel RLC circuit model.

Comparison of RESON simulation and test results.

| Parameter | MODEL | TEST |
|---|---|---|
| f0 [MHz] | 53.69 | 59.48 |
| Lp [uH] | 0.76 | 0.74 |
| Cp [pF] | 9.7 | 9.7 |
| Rp [kOhms] | 236 | 220 |

The results are fairly good for this geometry, but other tests with coils of oval crossection placed in a rectangular shield show larger errors. The reason for this can probably be attributed to the modeling of the lumped capacitance and the shield reduction factor. These numbers are difficult to accurately model in strange, non-coaxial shapes. Using some other tools like POISSON or empirical relations in conjunction with this analysis method would help expand the design rules for various geometries and improve the accuracy of the results.