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Tim Berenc
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CCP K500 Tuning Stem Design

Introduction

The purpose of this RF note is to model, analyze, and design the Tuning Stem of the K500 resonators for the Coupled Cyclotron Project (CCP). The theory behind the analysis method is briefly explained and applied to an analysis of the original K500 resonators for which both previous analytical and experimental data exist. The comparison between the presented analytical data and the previous data will give a measure of the method's error in predicting the performance of the CCP K500 Dee Resonators. Finally, the model is used to explore the characteristics of various stem designs from which a final design is found and analyzed.

The Modeling Method

The method applied to modeling the cyclotron resonators has been conceptualized by John Vincent and can be found in his dissertation defended at Michigan State University [1]. Overly generalized, the method models two-conductor RF structures using a lossy TEM transmission line approach while compensating for longitudinal field effects, due to discontinuities and non-uniformities, with the conservation of energy.

The modeling theory begins with the well known theory of a uniform lossless transmission line. Transmission line theory is based upon the TEM mode for which the voltage and current are directly related to the electric and magnetic fields. This is true since the fields exist only in the plane transverse to wave propagation. The TEM mode can only exist within a two conductor system or a one conductor system provided that there is a beam current within the conductor. The unique property of the TEM mode is that its group velocity along the guided wave axis is not dependent upon frequency and is only dependent upon the permeability and permittivity of the medium as given by the equation:

$$\upsilon = \frac{1}{\sqrt{\mu\varepsilon}} \qquad (1)$$

where μ and ε are respectively the permeability and permittivity. This implies that the TEM mode has no cutoff frequency, a property which allows for a transmission line to be characterized by an electrostatic solution of it's electrical parameters.

Since the voltage and current are directly related to the electric and magnetic fields respectively, the voltage and current propagate with the same velocity as the TEM mode. Based upon a distributed circuit model for a TEM mode transmission line, the velocity is also given by:

$$\upsilon = \frac{1}{\sqrt{\ell C}} \qquad (2)$$

where ℓ is the inductance per unit length and C is the capacitance per unit length. The voltage and current can be related through a characteristic impedance Z_o as:

$$\frac{V(z,t)}{I(z,t)} = Z_o = \sqrt{\frac{\ell}{C}} \qquad (3)$$

From (1) and (2), this characteristic impedance can then be expressed as:

$$Z_o = \frac{1}{\upsilon \cdot C} \qquad (4)$$

with $\upsilon = \frac{1}{\sqrt{\mu\varepsilon}}$. The capacitance, C, between two conductors is given as the ratio of the induced charge per unit length on either conductor to the potential difference between the conductors as

$$C = \frac{|Q|}{V} \quad . \qquad (5)$$

This ratio is a constant that is only geometrically dependent. In order to obtain a value for this capacitance, one of the conductors can be raised to a potential of V and the induced charge on either conductor then calculated from a knowledge of the field solution at the surface of the conductor. The induced charge is determined from an integration of the normal component of the electric field over the surface of the conductor as given by Gauss' Law,

$$Q = \oint_S \varepsilon \cdot \vec{E} \bullet d\vec{S} \ . \qquad (6)$$

Accounting for Conduction Losses

The modeling method accounts for conduction losses due to non-uniform current distributions by mapping these non-uniform distributions to an equivalent uniform current distribution. The non-uniform current distributions arise in the K500 because the dee and liner are not concentric cylindrical conductors. The dee and liner are constructed such that the electric and magnetic fields are concentrated near the median plane of the Dee cross sections which in turn causes a current concentration in the same location. For a clearer understanding, view the field solutions for the various cross sections shown in figures 3-6.

Mapping this non-uniform current distribution into an equivalent uniform current distribution results in the expression for an equivalent width of conductor over which the current is uniformly distributed. It is obtained by realizing that the surface current is proportional to the magnetic field which in turn can be expressed in terms of the electric field through Maxwell's equations. For details see Chapter 8 in [1]. The results are given as

$$w_e = \frac{\left[\int_0^L |E_n| dl\right]^2}{\int_0^L |E_n|^2 dl} \qquad (7)$$

where E_n is the normal component of the electric field on the conductor width, L.

Step Discontinuities

Step discontinuities along the guided wave axis are accounted for by including lumped energy storage elements which account for the stored energy associated with satisfying the boundary conditions at the local discontinuity. For a step in height in a parallel plate geometry, the stored energy is predominately electric and therefore may be modeled as a capacitor. A step in width in a parallel plate geometry results in predominately magnetic energy storage which is modeled as a lumped inductor at the local discontinuity.

For coaxial transmission lines, a step in either the inner and outer conductors results in predominantly electric energy storage which is modeled as a lumped capacitor at the discontinuity. This result was used repeatedly in the modeling of the K500 Dee Stem which has a tapered coaxial geometry. The tapers are modeled by segmenting the transmission line into an appropriate number of segmentations (to be explained shortly) and accounting for the taper as a coaxial discontinuity.

Transmission Line Segmentation

The method allows for non-uniform transmission lines to be segmented along the guided wave axis into *n* transmission lines. The n^{th} transmission line parameters may be determined based upon cross sections taken at the two ends of the segment. Any discontinuities or non-uniformities are accounted for by including lumped energy storage elements at the discontinuity.

K500 Dee Resonator Model

In order to predict the behavior of the K500 Dee Resonators, an appropriate model was developed based upon the above mentioned method. The purpose of the K500 Dee Resonators is to supply energy to the beam in the form of an accelerating electric field force along the beam path. Since it is desired to have the beam path lie entirely in a plane, the K500 Dee Resonators should supply an electric field only in a direction tangential to this plane. To accomplish this, the resonating structures are made symmetric about the median plane, resulting in no electric field component crossing the median plane. Also, to maintain the largest electric field at the median plane, it is obvious that the resonator should be a ½ wave resonator resulting in a voltage maximum (current minimum) at the median plane.

To achieve a half wave resonator, a half wavelength transmission line with shorted ends is used while being electrically excited through capacitive coupling at a location near to the voltage maximum. The input coupling is used only on the lower half of the dee resonators causing a slight asymmetry which results in an electric field crossing the median plane. However, since the dee halves are tied together along the outer radius of the dee and at the tip, this electric field is relatively small. The ½ wave resonator can then be modeled as two separate ¼ wave resonators mated together.

Dee Model

Since the main goal of this project was to redesign the dee stems, a separate model of the dee and the dee stem were formulated. First, the dee was modeled as follows:

1.) Segment the dee into cross sections lying in the field planes
2.) Perform an electrostatic analysis on each cross section to determine the electrical parameters resulting from considering each section as a uniform transmission line
3.) Determine an equivalent transmission line to account for non-uniformities along the dee from cross section to cross section
4.) Include appropriate models for the tip and tail capacitance as well as the Dee Fine Tuner and Input Coupler.

A top view of a K500 Dee is shown in figure 1 while it's equivalent circuit model is shown in figure 2. The transmission line parameters were determined based upon an electrostatic analysis of each cross section to determine its capacitance. The particular tool used in this case to determine the electrostatic field solutions was COSMOS/M Version 1.75. This package is a finite element analysis which solves for the electrostatic fields of an arbitrary boundary value problem. It allows for output of the solution's field components which are used as input to the program called COSPROJ, written by John Vincent. COSPROJ performs the surface integration of the normal electric field to obtain the induced charge of equation (6) allowing for the determination of the per unit length capacitance and therefore, the transverse characteristic impedance of each cross section. COSPROJ also calculates the equivalent widths discussed in the theory section onto which the dee conductor and liner conductor are mapped.

Generalized, the cross sectional analysis was performed as follows:

1.) Perform 2D electrostatic field solution with COSMOS (or appropriate analysis tool) based upon the dee conductor being raised to a potential of 1V. This potential is used as a means of mathematical convenience in solving for the capacitance and hence the transverse impedance per unit length.
2.) Output the results into the appropriate format for input into COSPROJ
3.) Obtain the results for the transverse characteristic impedance, Z_t , and equivalent widths, w_e , from COSPROJ
4.) Proceed on to model the transmission line taking into account the non-uniformity of the dee (to be discussed shortly)

The specific modeling procedure used for COSMOS can be found in Appendix A.

The cross section electrostatic potential solutions for each of the cross sections taken along the dee as shown in figure 1 can be found in figures 3-6.

To model the transmission lines connecting the various cross sections along the dee, the method applied uses a non-uniform transmission line modeling technique. Assuming that the cross sections are segmented small enough such that the transverse characteristic impedance, Z_t, is varying linearly, the transmission lines can be characterized from a knowledge of the transverse characteristic impedances at the end cross sections. The equivalent transverse characteristic impedance of the transmission lines connecting the cross sections is given by [1] as

$$Z_t = Z_{t1} \cdot \sqrt{\frac{(\tau^2 - 1)}{2\ln(\tau)}} \qquad (8)$$

where $\tau = \frac{Z_{t2}}{Z_{t1}}$ and Z_{t1} and Z_{t2} are the transverse characteristic impedances of cross-section 1 and 2 respectively. Equation (8) is developed by minimizing the error in conserving the electrostatic and magnetostatic energy along the segment.

The conductor losses are accounted for by preserving the losses along the line as the equivalent conductor widths change. Assuming that these equivalent conductor widths also change linearly along the line, which is a valid assumption if the segments are taken short enough, the equivalent conductor width for each conductor of the transmission line is given by [1] as:

$$w_e = \frac{w_{e2} - w_{e1}}{\ln\left(\frac{w_{e2}}{w_{e1}}\right)} \qquad (9)$$

where w_{e1} and w_{e2} are the equivalent widths of a conductor at the cross sectional ends of the transmission line. The material losses of the intervening medium between the two conductors are handled through a specification of a quality factor for the isotropic, linear dielectric and permeable material.

Based upon the above formulation, the K500 dee transmission line representation can be completely characterized from the cross sectional parameters given in figures 3-6. The transmission line parameters are shown for convenience in figure 1.

Dee Fine Tuner Model

The capacitance between the dee surface and the dee fine tuner was calculated by finding the induced charge on the trimmer plate when the Dee was raised to a potential of one volt. This induced charge is a direct measurement of the capacitance per unit length that the Dee Fine Tuner adds to the overall circuit. The total capacitance is found by multiplying through by the overall length over which the Dee Fine Tuner extends along the Dee Surface. The induced charge obtained was 47 pF/m resulting in a total capacitance of 6.8 pF due to the total length of the tuner along the direction of wave propagation.

The conductor that ties the Dee Fine Tuner to the Liner must also be modeled. This was done by performing an electrostatic analysis of a cross sectional view taken parallel to the median plane at the location of this conductor. The cross section view showing the equipotential contour along with the transmission line parameters obtained are shown in figure 7.

Input Coupler Model

A cross sectional view of the input coupler protrusion into the Dee is shown in figure 8. The input coupler is an electric field probe which uses capacitive coupling to couple energy into the resonator while matching the transmission line from the transmitters to the impedance of the dee resonator as seen at the location of the coupler.

Matching impedances is used to eliminate any reflections from occurring at the junction between two unmatched impedances. This basically means that the energy delivered by a source is entirely absorbed by the load. In terms of power, which is the term conventionally applied in circuit theory, the power (or energy flow) supplied by the source is entirely dissipated within the load.

If coupling is achieved with reactive elements at any point within a resonator, the condition at an impedance match is as follows:

$$\frac{V_l^{\,2}}{Z_o} = \frac{V_r^{\,2}}{R_S} \qquad (10)$$

where V_l is the line voltage on the feed line, Z_o is the characteristic impedance of the feed line, V_r is the voltage of the resonator at the coupling point, and R_S is the equivalent shunt resistance of the resonator at the coupling point. Equation (10) is the operating condition at an impedance match. Since the feed line will see an impedance equal to its own characteristic line impedance, the source supplies a power equal to the left hand side of equation (10).

Since no real power is dissipated within the reactive elements used to achieve a match, this supplied power is completely dissipated within the resonator and is expressed as the right hand side of equation (10). The reactive elements used to achieve a match basically provide a voltage drop which will cause the condition of equation (10) to be satisfied.

Physically, the coupling method used here at NSCL is capacitive coupling. In this method, a capacitive reactance provides the voltage drop to satisfy the condition of equation (10). The actual value of capacitance needed to achieve this impedance match at resonance is derived in [1] and is calculated within the circuit analysis program WAC. The addition of this capacitive element does cause a slight resonant frequency shift due to the addition of its reactance to the resonator.

In particular the method used to determine this value of input coupling capacitance went as follows:

1.) Model one half of the entire dee resonator, which is a ¼ wave resonator consisting of the dee and the dee stem
2.) Find the tuning stem position for the desired resonant frequency
3.) Determine an equivalent parallel RLC circuit for the circuit response of step 2 and place it at the tip of the dee. This allows for including the other half of the dee structure as a mating ¼ wave resonator located at the tip of the dee
4.) Next, determine the equivalent parallel RLC circuit at the desired point of coupling
5.) Calculate the capacitance value needed to achieve an impedance match for a line with characteristic impedance Z_o at this coupling point

Steps 4 and 5 are actually done within WAC. All the user has to do is supply the information as to the point of coupling and the characteristic impedance of the line. Once the capacitance value is returned to the user from WAC, this capacitance must then be added to the circuit model at the point of coupling.

There is another capacitance which is introduced by the existence of the cylindrical extension coming off of the dee surface as shown in figure 9. This capacitance was calculated by finding the difference in the stored electric energy between an electrostatic analysis of the cross section with and without this protrusion. The electric energy is related to the capacitance through the relation

$$U_e = \frac{1}{2} C V^2 \qquad (11)$$

An analysis of the cross section with the protrusion results in a larger electric energy storage than without the protrusion. The capacitance per unit length needed to make up for this difference is given as

$$C_d = 2 \cdot (\Delta U_e) \qquad (12)$$

where ΔU_e is determined with a potential difference of 1 volt applied to the conductors. Therefore the capacitance accumulated due to the length along the Dee surface (as traveling along the direction of wave propagation)is estimated by assuming a total length equal to the radius of the cylindrical protrusion. This gave a total capacitance of 0.194 pF.

Tip And Tail Capacitance

The capacitance between the dee and liner conductors at the tip and tail of the dee were also modeled. The tail capacitance represents an approximate parallel plate geometry and may therefore by modeled with the standard capacitance formula for such a geometry

$$C = \frac{\varepsilon \cdot A}{d} \quad \text{Farads} \qquad (13)$$

where A is the area of one plate and d is the distance between the plates. The dimensions used for calculating this capacitance were a width, length, and separation distance of 2.36 cm. The resultant capacitance value representing the tail of the dee was 0.209 pF.

Determining the value for the tip capacitance was a bit more complicated since the tip of the dee resonator is a complex geometry. A cross sectional view is shown in figure 10. A first order type approximation for the capacitance of such a geometry is to use standard parallel plate and coaxial capacitance formulas. The lower left view on figure 10 shows the locations for the parallel plate approximations while the lower right view shows an example location for the coaxial approximations.

Since the cross sectional geometry of the tip is spanned cylindrically about the center of the cyclotron, the parallel plate and coaxial capacitance formulas are modified by a factor corresponding to the degree of cylindrical rotation. The degree of rotation for each capacitive section is shown in the lower right view of figure 10. In this fashion, the approximation for the tip capacitance was found to be 2.15 pF.

Dee Stem Model

The dee stem is a non-uniform coaxial shorted transmission line. It was modeled using the method of tapers found in [1]. In order to segment the tapered section appropriately such that the transverse impedance, Z_t, does in fact vary linearly we need to observe the behavior of Z_t along the taper. There are three taper cases assuming that the taper itself is a linear taper:

1.) The inner conductor diameter varies while the outer conductor diameter remains constant
2.) The outer conductor diameter varies while the inner conductor diameter remains constant
3.) Both the inner and outer conductor diameters vary

These cases can be considered by noting the expression for the transverse impedance of a coaxial line,

$$Z_t = \frac{1}{2}\sqrt{\frac{\mu}{\varepsilon}} \cdot \ln\left(\frac{b}{a}\right) \qquad (14)$$

where *a* and *b* are the inner and outer radii respectively. If for case 1 and 2, *a* and *b* are assumed to vary linearly, Z_t can be considered to vary linearly over a segment in which the slope $\frac{dZ_t}{dL}$ goes to zero. For this to be the case, it ends up that the taper should be segmented in lengths that are much smaller than the ratio of the inner (outer) radius to the slope of the taper of the inner (outer) radius, expressed as,

$$L << \frac{r}{slope} \qquad (15)$$

For the case in which both the inner and outer conductor radii vary linearly with the same slope, the transverse impedance also varies linearly. Therefore, tapers with both radii varying should be segmented into sections over which the taper can be assumed linear.

Longitudinal field effects are accounted for by including a lumped coaxial step discontinuity capacitance. For tapered sections over which both inner and outer radii are changing, the shunt capacitances for the inner (outer) conductor tapers were calculated by holding the outer (inner) conductor at a fixed radius equal to the average of its radius at the ends of the segment.

Besides modeling the stem as a coaxial tapered line, pure coaxial step discontinuities also occur along the stem. These step discontinuities are modeled as shunt capacitances. Also, resistances due to the ringed annuli which exist at these step discontinuities are calculated directly within the circuit analysis program by inputting them as an annulus circuit element.

Contact Fingers and Bolted Joints

The contact fingers were modeled as resistances whose values were obtained from a knowledge of the circumference of the fingered ring along with the resistance of these fingers on a per width basis as determined in [1]. The bolted joints which connect various sections of the dee stem were modeled as resistances whose values were obtained in a similar manner to the fingers. Details can be found in [1]

Circuit Analysis Program

Once all the electrical parameters for each element was determined, a transmission line circuit analysis had to be performed to obtain the overall circuit response. The transmission line analysis program used for this analysis was provided by Precision Power Products (www.3pco.com). The program is called WAC which stands for Windows A.C. The program is a complete lossy transmission line analysis program.

In particular, the transmission line analysis program was used as a means to determine the resonant frequency of the dee resonators as a function of the stem's short position as well as the voltages, currents, and dissipated power for the entire dee resonator.

Original K500 Dee Stem Analysis

Since previous analytical and experimental data already exist for the original K500, its analysis would provide great insight into the accuracy of the modeling technique. Since the dee circuit model has already been described, all that is needed to complete the resonator model is the model of the original dee stem. The original dee stem is pictured in figure 11 as well as its transmission line segmentation parameters. The additional reactive elements added to the Stem circuit model due to tapers and discontinuities are given below

Table I

Taper and Discontinuity Capacitances

Name	Location	Type	Value (pF)
CT_1	TR_ST2a - TR_ST2b	Taper	0.272
CS_1	TR_ST3 - TR_ST4	Discontinuity	0.267
CS_3	TR_ST5 - TR_ST6	Discontinuity	0.082
CS_4	TR_ST6 - TR_ST7	Discontinuity	3.11
CS_5	TR_ST7 - TR_ST8a	Discontinuity	3.03
CT_2	TR_ST8a - TR_ST8b	Taper	0.21
CS_6	TR_ST8b - TR_ST9a	Discontinuity	0.261
CT_3	TR_ST9a - TR_ST9b	Taper	0.323
CT_4	TR_ST10a - TR_ST10b	Taper	0.974
CS_7	TR_ST10b - TR_ST11a	Discontinuity	1.69
CT_5	TR_ST11a - TR_ST11b	Taper	0.567
CT_6	TR_ST11a - TR_ST11b	Taper	0.533
CT_7	TR_ST12a - TR_ST12b	Taper	0.126
CT_8	TR_ST13a - TR_ST13b	Taper	0.101
CT_9	TR_ST14a - TR_ST14b	Taper	0.078
CT_10	TR_ST15a - TR_ST15b	Taper	0.064
CS_8	TR_ST16 - TR_ST17	Discontinuity	5.07
CS_9	TR_ST17 - TR_ST18a	Discontinuity	7.06
CT_11	TR_ST18a - TR_ST18b	Taper	0.45
CT_12	TR_ST19a - TR_ST19b	Taper	0.143
CT_13	TR_ST20a - TR_ST20b	Taper	0.109
CT_14	TR_ST21a - TR_ST21b	Taper	0.107
CS_10	TR_ST21b - TR_ST22	Discontinuity	0
CS_11	TR_ST22 - TR_ST23	Discontinuity	0.462

The resistive elements added to the original Dee Stem circuit model due to annuli, bolted joints and the fingers at the short are given below:

Table II

Annuli Dimensions

Name	Location	R_i (cm)	R_o (cm)
AN_1	TR_ST3 - TR_ST4	10.06	12.07
AN_2	TR_ST4 - TR_ST5	10.06	10.16
AN_3	TR_ST5 - TR_ST6	10.16	11.11
AN_4	TR_ST6 - TR_ST7	5.24	9.21
AN_5	TR_ST7 - TR_ST8	11.11	16.03
AN_6	TR_ST10 - TR_ST11	17.46	21.07
AN_7	TR_ST16 - TR_ST17	15.24	22.23
AN_8	TR_ST17 - TR_ST18	12.21	22.23
AN_9	TR_ST21 - TR_ST22	25.48	27.61
AN_10	TR_ST22 - TR_ST23	18.25	25.48
AN_Short	TR_ST23 - COM	5.24	18.25

Table III

Bolted Joints and Fingers

Name	Location	Type	R (Ohms)
R_J1	TR_ST1 - TR_ST2	inner bolt	1.24E-04
R_J2	TR_ST3 - TR_ST4	outer bolt	4.54E-05
R_J3	TR_ST5 - TR_ST6	outer bolt	6.27E-05
R_J4	TR_ST6 - TR_ST7	inner bolt	1.22E-04
R_J5	TR_ST7 - TR_ST8	outer bolt	5.72E-05
R_J6	TR_ST21 - TR_ST22	inner bolt	1.04E-04
R_J7	TR_ST22 - TR_ST23	outer bolt	2.50E-05
R_J8	TR_ST23 - COM	outer fingers	1.31E-04
R_J9	TR_ST23 - COM	inner fingers	4.56E-04

A representation of the circuit model for the original Dee Stem can be found in figure 12.

There is one fact left to be noted for the modeling of the dee stem. The transmission line represented by the sliding short segment consists of an outer conductor that is composed of panels which are pieced together into a hexagonal shape. Therefore, this line is not a true coaxial geometry. In order to map this line to an equivalent coaxial geometry, the inner conductor was kept equal to its original dimension while the outer conductor radius was altered to give the same effective stored electric energy or capacitance. This was achieved by determining what the electric stored energy was for the hexagonally shaped cross section with COSMOS. Due to symmetry, only 1/3 of the hexagon needed to be modeled. A cross sectional view including the electric field lines can be found in figure 13.

From COSMOS, the stored electric energy was found to be 7.23 pJ. Therefore, the total energy is 21.69 pJ, three times this value. Since 1 volt was used for the solution to the boundary value problem, this energy value corresponds to a capacitance of 43.38 pF, twice the energy value. Using the equation for the capacitance per meter of a coaxial line given as

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)} \qquad (16)$$

we can solve for the value of b that will give an equivalent value of capacitance. With a fixed at the inner conductor radius of 2.0625 inches, b is found to be 7.43 inches. The value of the characteristic impedance is 76.84 Ohms. The transmission line parameters for this last transmission line of the dee stem have already been included in the transmission line segmentation view of figure 11.

The overall circuit representation of the resonator as shown previously in figure 2 was simulated within WAC as follows:

1.) First, a ¼ wave representation of the resonator was simulated varying the sliding short length to tune the resonator to the desired frequency for analysis.
2.) Once the sliding short was tuned for the appropriate frequency, the equivalent parallel RLC circuit representation of the ¼ wave resonator at the Dee tip was determined within WAC.
3.) The equivalent parallel RLC circuit was then mated to the ¼ wave resonator to represent the other half of the resonator, or the mirrored ¼ wave resonator.
4.) The circuit was again simulated noting that the resonant frequency found in step 1 was not altered.
5.) Next, the input coupling capacitance needed to achieve an impedance match to the feed line at the point of the input coupler (between transmission lines TR_D10 and TR_D11) was determined within WAC by specifying the characteristic impedance of the feed line and the node at which coupling was desired.
6.) The capacitance determined in step 5 was then added to the circuit as C_IC in figure (2). The circuit was again simulated while adjusting the sliding short position in order to retune the circuit back to the original resonant frequency found in step 1 and 4.

A typical final WAC circuit input file can be found on the preceding pages. The file is representative of the final circuit that includes the effective mating resonator and the input coupler.

Accuracy of Original K500 Dee Stem Model

To determine the accuracy of the current modeling technique, the K500 with the original dee stem was analyzed and compared with previous physical measurements. The criterion used for the comparison was a measurement of the short position at the extremes of the proposed frequency range of operation for the CCP K500. The operating frequency range design objective for the CCP K500 is 11 MHz to 27 MHz. Therefore, the current analysis results were compared to the measurements at these two frequencies.

An initial analysis at 11 MHz yielded a stem position of 71.6 inches while previous experimental measurements gave a result of 76.30 inches. At 27 MHz, our initial analysis yielded a stem position of 3.24 inches while previous experimental measurements gave a result of 2.80 inches. After reviewing the dee resonator model, I noticed that I had forgotten to include transmission lines that would connect the tip and tail to the very end cross sections A and P of figure 1. The parameters for these transmission lines were taken to be the characteristics of the last cross sections taken, namely A and P whose electrostatic analysis results can be found in figures 3 and 6.

Including these transmission lines resulted in an even shorter stem position prediction at 11 MHz. We therefore investigated our dee resonator model further to search for other sources of error. It was discovered that the capacitance formulas for the coaxial step discontinuities and tapers along the stem were incorrect. Correcting these capacitance values (note the corrected values are the ones given in Table I) resulted in an 11 MHz resonance prediction at a short position of 82 inches. We went from an under prediction to an over prediction of the short position.

Once again, we reviewed our resonator model and set out to find the error source. The major source of error was found at the top of the corona ring. If you review the picture of the original dee stem, you will notice that the top of the corona ring extends rather far under the outer conductor. It was possible that the taper and step discontinuity capacitances do not completely account for the capacitance seen at this location. To investigate the problem, we performed an electrostatic analysis of the entire corona.

Electrostatic Analysis of the Original Dee Stem Corona

The electrostatic analysis package COSMOS allows for the simulation of axially symmetric geometries. Therefore, the entire corona region of the original dee stem resonator was simulated and the stored electric energy calculated. To determine whether our model accounted for the total capacitance along this region, I accrued the capacitance represented by all the transmission lines and discontinuities along this region and compared it with the value of capacitance required to obtain the stored electric energy as calculated by COSMOS. The total accumulation of capacitance represented by our initial model was found to be 123 pF.

A view of the equipotential contours as found by COSMOS is shown in figure 14. For axially symmetric geometries, COSMOS outputs the stored electric energy in a per radian value. Therefore, the value obtained had to be multiplied by 2π to obtain the entire stored energy. With a potential difference of 1 volt between the inner and outer conductors, the stored energy as determined from the COSMOS calculation was found to be 65.03 pJ. This represents a capacitance value of 130.06 pF, a difference of 7 pF from our model.

Clearly, our initial model did not accurately account for the capacitance near the top of the corona. This excluded capacitance was then added between transmission lines TR_ST17 and TR_ST18 shown in figure 11. Once again simulating the model with WAC, the 11 MHz resonance was predicted at a short position of 79.09 inches and the 27 MHz resonance was predicted at 3.30 inches. This is only a 3 inch error at 11 MHz and a 0.5 inch error at 27 MHz. These errors were quite acceptable and give us confidence in our dee model for the analysis of a new design. However, these errors should be kept in mind when designing for error tolerances.

To determine the error allowance in the design of the new stem, the resonator's sensitivity to the short length was observed at the extremes of the frequency range. At 11 MHz, a 1 inch extension in the short position yielded only a 72.5 kHz decrease in resonant frequency. At 27 MHz, a 1 inch increase in the short position yielded a decrease in the resonant frequency of 785 kHz, a little over 10 times the frequency sensitivity of the resonator at 11 MHz. Taking this frequency sensitivity data into account, we should allow for an excess length in the new stem design.

If we allow for an excessive 10-15 inches in the overall length of the stem, we are allowing for a safety margin of about 800 kHz at a resonant frequency of 11MHz. If we obtain resonance at 27 MHz with the short position at a point 3 inches from the bottom of the short panels, we are allowing for a frequency margin of about 2 MHz. The overall length of the CCP Tuning Stem panels is only 101 inches. Therefore, if we meet the design objective of an operating frequency range between 11 MHz to 27 MHz, we should obtain 11 MHz resonance at about 85-90 inches while obtaining 27 MHz resonance at 3 inches. Note that these lengths are measured from the bottom of the tuning stem panels as the total length of the final transmission line of the stem model.

CCP Proposal Stem Design Analysis

Before I arrived at NSCL, a CCP K500 stem design proposal had already been presented. Therefore, I proceeded to analyze this proposed design. The design is shown in figure 15 along with its transmission line segmentation.

The analysis resulted in an 11 MHz resonance at a short position of 98.66 inches and a 27 MHz resonance at a short position of 2.74 inches. This design would allow us to obtain resonance at 27 MHz with a reasonable frequency margin. However, considering that the total stem length is only 101 inches and our previous model error was 3 inches at 11 MHz, this design would not a allow a reasonable error margin at 11 MHz.

Final Stem Design

It was noted that in order to achieve resonance at 11 MHz with the short position between 85 - 90 inches, we would have to add more capacitance to the stem. In order to do this we could increase the inner diameter of transmission line TR_ST3 of the CCP Proposal Stem Design shown in figure 15. By changing the characteristic impedance of this line directly within WAC, I found a that a characteristic impedance value of 22.5 Ohms would result in an 11 MHz resonance at a short position of about 86 inches while minimally affecting the short position at 27 MHz.

Knowing this, I first attempted to extend the taper from the corona ring down to the top of the Dee. Although, this increased the capacitance along that length, resonance at 11 MHz still occurred at a somewhat excessive short position of 94 inches. This would give us only a 7 inch tolerance since the total length is only 101 inches. Considering the 3 inch error in the original predicted short position, the design margin was too close. The reason why this design did not work was that I used a taper that didn't maintain an effective constant impedance of the calculated 22.5 Ohms.

Consequently, I decided to maintain the calculated characteristic impedance of 22.5 Ohms along the transmission line TR_ST3 with a non-tapered section. This was achieved by increasing the radius of the inner diameter along this section in accordance with the equation for the characteristic impedance of a coaxial transmission line as given by

$$Z_O = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{b}{a}\right) \qquad (17)$$

where b is the outer conductor radius and a is the inner conductor radius. For a characteristic impedance of 22.5 Ohms and *b* fixed to its previous dimension of 4.375 inches, *a* must be 3 inches.

Before analyzing this design, I decided to investigate the chances of sparking in the region where the liner blends into the stem around TR_ST2 of the CCP Proposal Stem as shown in figure 15. Sparking occurs within vacuum under DC conditions when an electric field equal to 100 kV/cm at the conductor surface is reached. Considering a coaxial line with an inner radius equal to 3 inches and an outer radius equal to 4 inches (the outer diameter of the stem at TR_ST2 of the CCP Proposal Stem) and the expression for the electric field between the conductors of a coaxial line given as

$$\vec{E} = \frac{V}{r}\cdot\frac{1}{\ln\left(\frac{b}{a}\right)} \qquad (18)$$

the maximum electric field with V equal to 100 kV would be 45.62 kV/cm at r equal to *a*. This value is less than ½ the value at which sparking occurs. The voltage value used in determining this maximum field was chosen from the maximum foreseeable dee voltage at which the cyclotron would be operated. Since this region is relatively close to the voltage maximum, it would exhibit a voltage near to this maximum.

Since the event of a spark was ruled out, I proceeded with the design. The taper near the vacuum window needed to be altered to provide a smooth transition into the calculated inner diameter radius of 3 inches. This final design is shown in figure 16 along with its transmission line segmentation.

Electrostatic Analysis of the Corona Region

Similar to the analysis of the original dee stem, an electrostatic analysis of the corona region was performed to compensate for the capacitance at the top of the corona.
An equipotential contour plot of this analysis as performed by COSMOS can be found in figure 17. The total capacitance accrued from our model without the added capacitance was found to be 72.3 pF. The total stored electric energy as calculated by COSMOS was found to be 37.18 pJ corresponding to a capacitance of 74.36 pF. Our initial model therefore did not account for 2.06 pF of capacitance due to the top of the corona. Our model was then corrected by adding a 2.06 pF shunt capacitor between transmission lines TR_ST13 and TR_ST14 as shown in figure 16. This value of capacitance was much smaller than the additional capacitance that needed to be added to the original dee stem because of the difference in the two corona geometries as seen in the equipotential contours of figures 14 and 17.

An analysis of this final design resulted in an 11 MHz resonance at a short position of 85.24 inches and a 27 MHz resonance at a short position of 3.24 inches. Clearly this design will meet the operating frequency range design objective with a sufficient error tolerance in the stem length at both extremes of the frequency range. A comparison of the various designs at the extremes of the frequency range can be found on the proceeding page. Following that is a typical WAC input file for the final stem design.

Following is a list of the results from the final circuit simulations:

Table IV

Frequency (MHz)	Q	P (kWatts)	Short Position (in)
11.0053	4232	50.59	85.24
13.0024	4133	55.71	59.37
15.0042	4080	62.01	42.22
17.0043	3971	68.44	30.31
19.0045	3827	75.58	21.71
21.0023	3659	83.02	15.28
23.0045	3483	90.66	10.33
25.0047	3305	98.7	6.65
27.0082	3001	106.8	3.24

The dissipated power is normalized to a dee voltage of 100 kV peak.

A complete characterization of the final stem design in terms of frequency versus short position is shown in figure 18. Figure 19 represents the total dissipated power normalized to a dee voltage of 100 kV peak. Continuing on, figures 20 and 21 represent the dissipated power distribution along this final stem design. This information is used by the mechanical group in order to design the water cooling system. Information which is pertinent to the physicists is the voltage distribution along the Dee inner and outer surfaces as a function of the radial distance from the center of the cyclotron. This information can be found on pages 50 to 55.

References

[1] J. Vincent, "Modeling and Analysis of Radio Frequency Structures using an Equivalent Circuit Methodology with Application to Charged Particle Accelerator RF Resonators", Michigan State University, Department of Electrical Engineering.

Appendix A

COSMOS Modeling Procedure

1.) Open a new file
2.) Load in the DXF file of the cross section from the Control panel
3.) Define the Liner as contour #1
4.) Define the Dee as contour #2
5.) Define Region #1 as a two contour region consisting of contours 1 and 2
6.) Mesh the region using the 'quad-mesh surface/region' function or a mesh with which the user feels comfortable. However, for electrostatic field solutions with a finite element method, a quadrilateral mesh must be used.
7.) Define the boundary conditions be placing 1 volt on the Dee contour and 0 volts on the Liner contour. Then erase the boundary condition of 0 volts on the curve at the median plane. This is the curve about which there is symmetry. The symmetry condition for FE algorithms is no boundary condition.
8.) Run the electrostatic solver
9.) Select all nodes along the dee and liner surfaces and list the resultant electric fields to an output file to be used by COSPROJ
10.) Select only the dee surface contour and output the node numbers to an output file to be used by COSPROJ
11.) Select only the liner surface contour and output the node numbers to an output file to be used by COSPROJ.
12.) Using the output files created in steps 9-11, prepare three separate database files entitled, 'deenodes.dbf', 'linnodes.dbf', 'fields.dbf' using an appropriate spreadsheet program.
13.) Run COSPROJ inputting one of these files.
14.) View the output of COSPROJ to obtain the equivalent widths and characteristic impedance of the cross section.