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K500 Dee Voltage Pickup-Loop Scaling Factor and Phase Comparison Investigations

Introduction

In order to measure the peak dee-voltage of a dee-resonator and the phase relationship between resonators, the K500 cyclotron utilizes inductive pickup-loops located on the resonator sliding shorts. The peak dee-voltage is indirectly related to the emf induced onto a pickup-loop through the geometry of the resonator and pickup-loop. Furthermore, the orientation of the pickup-loop determines the phase relationship between the induced emf and the dee-voltage. By preserving this phase information through time-delay matched cables, the resonator-to-resonator phase relationships are directly measured by the phase between a pickup-loop from each resonator.

This note presents the comparison between the theoretical and experimental scaling factors relating the induced emf on a pickup-loop to the dee-voltage. The theoretical scaling factor is determined from the geometrical problem while the experimental scaling factor is determined from x-ray measurements of the dee.

The accuracy of the scaling factor is of extreme practical value. The accuracy of the scaling factor gives a measure of the uncertainty to which the dee-voltage of a particular resonator is known. Furthermore, associated with this uncertainty is a measure of the imbalance between any two resonator dee-voltages. This measure of imbalance is of interest because all resonator dee-voltages should be identical for an ideally tuned cyclotron.

Similarly, the accuracy of the phase measurement between resonators is also of practical interest. Again, this accuracy gives a measure of the uncertainty to which the phase between any two resonators is known.

This note presents the results of an investigation of the voltage and phase accuracy of the dee pickup-loops.

The Theoretical vs. Experimental Scaling Factor

The scaling factor relating the emf induced onto the pickup-loop to the dee-voltage is determined from the geometry of the resonator and pickup-loop. Physically, there are two pickup-loops located on each dee-stem sliding short; thus totaling 4 pickup-loops for each resonator station. The representative geometry for a single pickup-loop is shown in Appendix A.

For the discussions of this note, the scaling factor, α , is defined as

$$\alpha = \frac{|V_{Dee}|}{|V_{Loop}|} \quad \text{(kVolts/Volt)} \qquad (1)$$

where V_{Dee} is the voltage at the resonator dee-tip and V_{Loop} is the voltage on the pickup-loop coax-cable due to the induced emf within the pickup-loop. Convenient units for this scaling factor are kVolts/Volt.

The governing equation which derives the mathematical relationship between the induced emf and the dee-voltage, is Faraday’s law:

$$V_{EMF} = -\frac{d\Phi}{dt} \qquad (2)$$

where Φ is the magnetic flux intercepted by the loop and V_{EMF} , measured in volts, is the electromotive force induced within the loop. The amount of magnetic flux, Φ , intercepted by the loop depends upon the geometry of the loop and the local magnetic field near the loop, which in turn is determined from the frequency-dependent current on the short.

Using equation (2), the magnitude of the induced emf within the pickup-loop is shown in Appendix A to be equal to the following formula:

$$|V_{EMF}| = \mu\, f\, K_G\, Y(f)\, |V_{Dee}| \qquad (3).$$

where μ is the permeability of the medium, $K_G = z \ln\left(\frac{\rho_2}{\rho_1}\right)$ is the geometric scaling factor based upon the loop geometry, and $Y(f)$ is a frequency-dependent scaling factor relating the current on the short to the resonator dee-tip voltage, V_{Dee}, at frequency f . In particular,

$$Y(f) = \frac{I_{short}(f)}{V_{Dee}} \qquad (4)$$

where I_{Short} is the current on the short at frequency f for the resonator dee-tip voltage, V_{Dee}.

Substituting equation (3) into equation (1) and using the relationship between V_{Loop} and V_{EMF} results in the scaling factor being equal to

$$\alpha = \frac{2}{1000 \cdot \mu f K_G Y(f)} \text{ (kVolts/Volt)} \qquad (5)$$

where the factor of 1000 is used such that α is in (kVolts/Volt) and the factor of 2 arises from a series 50 Ohm impedance at the actual pickup-loop creating a voltage divider with the 50 Ohm termination impedance on the pickup-loop RF coax-cable. The derivation of equation (5) can be found in Appendix A.

Using results from simulations performed in RF Note 116, the calculation of Y(f) is given in the following table

Magnitude of Current at the Dee-Stem Short for V_{Dee} = 100kV$_{PK}$ and Associated Y(f)

Freq (MHz)	I short (kA$_{PK}$)	Y(f) (x10^{-2})
11	2.287	2.287
13	2.537	2.537
15	2.796	2.796
17	3.035	3.035
19	3.264	3.264
21	3.473	3.473
23	3.659	3.659
25	3.824	3.824
27	3.964	3.964

For the K500 loop dimensions (ρ1=107.1mm, ρ2=110.5mm, z=3.429mm), $K_G = 1.08 \cdot 10^{-4}$.

Using the above theoretical data, the theoretical scaling factor, α, is pictorially represented below:

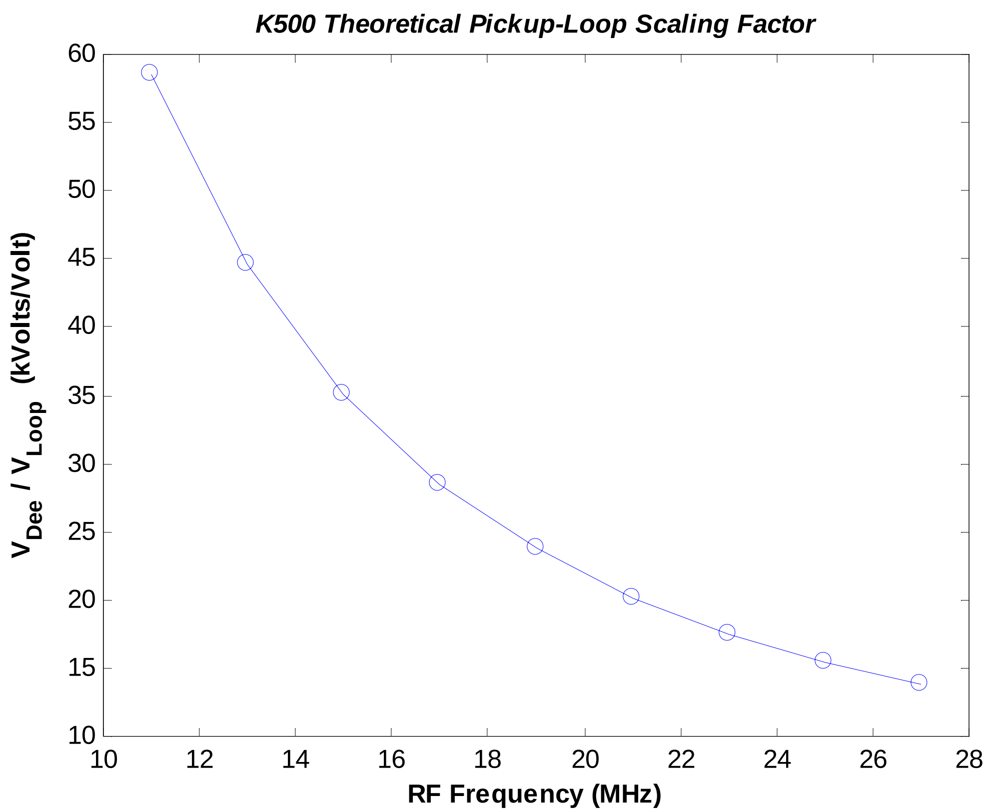


Figure 1: Theoretical Scaling Factor

Note that the scaling factor decreases as the frequency increases; clearly indicating that for the same dee-voltage, more voltage is induced onto the pickup-loop at higher frequencies.

The actual data used for the above graph is shown here:

Theoretical Scaling Factor vs. Freq.

Freq. (MHz)	V_{Dee} / V_{Loop} (kV/V)
11	58.52
13	44.64
13.3037	42.87
15	35.10
17	28.53
19	23.74
21	20.19
23	17.49
25	15.40
27	13.75

13.3037 MHz is included since this was the initial commissioning frequency.

Actual experimental x-ray measurement data is given below:

Experimental Scaling Factors from X-Ray Measurements

Frequency (MHz)	α_{ave} (kV/V)
13.3037	19.55
26.5	7.28

Comparing the experimental measurements to the theoretical data, it is seen that the theoretical scaling factor is greater by a factor of 2.19 at 13.3037 MHz and by a factor of 1.89 at 26.5 MHz. This means that in actuality a greater emf is being induced in the pickup-loop than we had calculated theoretically. This factor does not arise from an erroneous calculation with the voltage divider or from any errors in the theoretical derivation. These sources were carefully checked.

The source of error is most likely due to the fact that the theory does not take into account the non-ideal loop whose conductor's width is on the same order of magnitude as the width and height of its area. If the theoretical scaling factor is adjusted by the average of the discrepancy factor, the effective area of the pickup-loop can be calculated. The average of 2.17 and 1.88 is 2.04. This suggests that the geometrical factor, K_G, needs to be increased by a factor of 2.04. The effective height and width of the loop area can be increased equally by the dimension that solves the following formula:

$$(z+x)\cdot \ln\left(\frac{\rho_2 + x}{\rho_1 + x}\right) = 2.04 \cdot K_{G\ original}$$

Using the original value of $K_G = 1.08 \cdot 10^{-4}$ and the original loop dimensions, the solution to the above equaion was solved graphically as $x = 1.47mm$. Thus, increasing the height, z, by 1.47, increasing the outer radius, ρ_2, by 0.735mm and decreasing the inner radius, ρ_1, by this same 0.735mm, results in better matching the theoretical scaling factor to the experimental scaling factor. Thus, the 'effective' dimensions of the loop become, $z = 4.9mm$, $\rho_1 = 0.1063m$, $\rho_2 = 0.1112m$.

Adjusted Scaling Factor

Freq. (MHz)	Adjusted V_{Dee}/V_{Loop} (kV/V)
11	28.69
13	21.88
13.3037	21.01
15	17.21
17	13.99
19	11.64
21	9.90
23	8.57
25	7.55
27	6.74

Voltage & Phase Accuracy Investigations

In order to investigate the voltage and phase accuracy of the dee-voltage pickup-loops on the K500 cyclotron rf resonators, all three dee-tips were shorted together (thus forcing each dee to be at the same voltage) while each pickup-loop's induced voltage and relative phase were measured. The phase reference point was chosen to be one of the pickup-loops.

In order to excite the coupled dee-resonators, one transmitter was used as the source while the other two transmitters were de-coupled from the system. This de-coupling was achieved in a crude fashion by extracting the input coupler to its minimum capacitance and shorting the input coupling transmission line at the input coupler with the local spark gaps.

A crude measure of the effectiveness of this de-coupling method was a measurement of the directional coupler's reflected-power pickup. The reflected-power in this setup is an indication of how much energy is being coupled into the input-coupling transmission line and thus an indication of the degree of de-coupling. The peak-to-peak voltage level on the reflected-power pickup was measured to be less than 0.4 mV which is equivalent to only 0.4V pk-to-pk on the transmission line (directional coupler ratio is approx. –60dB). For comparison, it takes approximately 500 Volts on the transmission line to drive the dee-resonator to 25 kV at 13 MHz.

To further verify the degree of de-coupling, the amount of energy remaining on the transmission line was crudely measured by tuning the final anode tank circuit (the spark gap was not a perfect short) while monitoring the relative phase and amplitudes of the dee-voltage pickup-loops for any changes. No changes were discernible.

The effectiveness of shorting all three dees together was measured by monitoring the relative phase and amplitudes of the pickup-loops while each resonator was slightly mis-tuned. Again, no discernible changes were discovered.

These facts have been given not to confuse the issue, nor to raise intense debates as to the experimental setup. It would take a tremendous effort to theoretically validate the setup and measurements. This measurement was a first attempt at a means by which to gain some confidence in the voltage and phase measurements taken from the pickup-loops.

Ultimately, the data presented needs to quantify three things:

(1) the uncertainty of the dee-voltage value displayed in the control room,
(2) the imbalance between the dee-voltages of any two resonators,
(3) the uncertainty of the displayed phase-separation between the dees

These qualitative measures of the system are important factors when tuning the cyclotron beam. The data presented here quantifies these qualitative measures.

Pickup-Loop Voltage Comparisons

Ultimately, an average scaling factor relating the dee-voltage to the induced emf on the pickup-loops is determined. This scaling factor was previously defined as

$$\alpha_{ave} = \frac{V_{Dee}}{V_{Loop}}$$

where V_{DEE} is in peak kVolts and V_{Loop} is in peak Volts. The average will be an average of all pickup-loops of a particular station during an x-ray calibration of that station. It is important to note that this scaling factor is a function of frequency as shown previously.

The reason why an x-ray calibration was not done while the dees were shorted together was that the dee-voltage level was physically limited by the gap distances introduced by the dee-to-dee short. X-ray data is statistically inadequate at low voltage levels. No attempt was made to attain a high voltage with the short in place for fear of damaging the dee-tips and/or the central region.

The average scaling factor will be used for calculating a displayed readout value for the dee-voltage. Therefore, we are naturally led to relate all measured pickup-loop voltages to this average scaling factor. If, in actuality, a particular pickup-loop has a scaling factor, α_1, then the actual dee-voltage will differ from the displayed dee-voltage by P_1% of the displayed voltage; where

$$P_1 = \frac{V_{Dee1} - V_{Display}}{V_{Display}} \cdot 100 = \frac{\alpha_1 - \alpha_{ave}}{\alpha_{ave}} \cdot 100 .$$

Now, if all pickup-loop voltages are forced to be equal during tuning by forcing all the displayed voltages to be equal (remember: the average scaling factor is used for displaying all dee-voltages), then we now have a means to quantify the imbalance between each dee-voltage. If a second pickup-loop in actuality has a scaling factor, α_2, then its actual dee-voltage will differ from the displayed dee-voltage by P_2% of the displayed voltage; where P_2 is calculated in the same manner as P_1. Finally, the amount by which V_{Dee2} differs from V_{Dee1} can be calculated as

$$V_{Dee2} - V_{Dee1} = (P_2 - P_1)\% \ of \ V_{Display}$$

We have thus prepared a means by which to present the experimental scaling-factor data.

Currently, we have accumulated x-ray calibration data from a single resonator at only two frequencies, 13.3037MHz and 26.5MHz. The data at both frequencies were a result of running Station C. This data can be found in Appendix B.

X-ray data from the other two stations was inconclusive. The x-ray probe was at a disadvantageous position to see any x-rays from station B. Although x-rays from Station A were visible to the x-ray probe, the data accumulated was statistically poor at the attained dee-voltage of approximately 50kV at 13.3037 MHz. We had not yet commissioned the RF system before this data was taken; thus, 50kV was the maximum attainable dee-voltage on Station A at 13.3037MHz until we de-bugged the RF system. 50kV is on the threshold of statistically good x-ray data.

The results of the data in Appendix B yielded the following:

Experimental Scaling Factors from X-Ray Measurements

Frequency (MHz)	**α_{ave} (kV/V)**
13.3037	19.55
26.5	7.28

For future measurements, it is important to note that the above scaling factor represents the ratio between the dee-voltage and the voltage across a 50Ω load on a pickup-loop cable. The data was taken using a Fluke 85 rms voltage probe.

The scaling factor in the table above becomes the average scaling factor which will be used for displaying the dee-voltages on all stations. It is the α_{ave} of the previous discussion.

Now, the pertinent data from the measurements with all dees shorted together can be found on the following page.

Pickup-Loop Measurements at 15MHz with Dees Shorted

Pickup Loop	Station A Loop Voltage (V_{RMS})	Station B Loop Voltage (V_{RMS})	Station C Loop Voltage (V_{RMS})
DV1	0.740	0.738	0.768
DV2	0.751	0.756	0.757
DV3	0.745	0.742	0.760
DV4	0.754	0.748	0.744

Average of all Pickup Loops = 0.750

Pickup Loop	Station A % Diff. wrt ave.	Station B % Diff. wrt ave.	Station C % Diff. wrt ave.
DV1	-1.37	-1.63	2.37
DV2	0.10	0.77	0.90
DV3	-0.70	-1.10	1.30
DV4	0.50	-0.30	-0.83

Pickup-Loop Measurements at 23MHz with Dees Shorted

Pickup Loop	Station A Loop Voltage (V_{RMS})	Station B Loop Voltage (V_{RMS})	Station C Loop Voltage (V_{RMS})
DV1	0.771	0.757	0.792
DV2	0.760	0.780	0.773
DV3	0.778	0.756	0.778
DV4	0.764	0.769	0.763

Average of all Pickup Loops = 0.770

Pickup Loop	Station A % Diff. wrt ave.	Station B % Diff. wrt ave.	Station C % Diff. wrt ave.
DV1	0.12	-1.70	2.85
DV2	-1.31	1.29	0.38
DV3	1.03	-1.83	1.03
DV4	-0.79	-0.14	-0.92

The above data presents the percent differences of each pickup-loop voltage relative to the average voltage of all pickup-loop voltages. This was done since the average scaling factor is determined in a similar manner. This data cannot hastily be used to relate the percent difference of each loop's scaling factor relative to the average scaling factor, however. There is a subtle point given as follows:

Suppose we have two separate scaling factors, α_1 and α_2 expressed as

$$\alpha_1 = \frac{V_{Dee}}{V_{Loop1}} \quad \text{and} \quad \alpha_2 = \frac{V_{Dee}}{V_{Loop2}}.$$

where V_{Dee} is the same value. Now, if V_{Loop2} differs from V_{Loop1} by $L\%$ of V_{Loop1}, then α_2 will differ from α_1 by $P\%$ of α_1, where $P\%$ is given as

$$P\% = -L\% \cdot \frac{1}{\left(1 + \frac{L\%}{100}\right)}$$

This is exactly the situation we have presented in the above data. We have displayed the percent difference of each pickup-loop voltage with respect to the average pickup-loop voltage. But the quantity of interest which gives us the direct measure of the dee-voltage uncertainty is the percent difference of the scaling factors. This was discussed from the onset. Calculating $P\%$ for each pickup-loop from the data on the preceeding page yields:

Scaling Factor Percent Differences
α_N wrt α_{Ave} at 15MHz

Scaling Factor	Station A % Diff. wrt ave.	Station B % Diff. wrt ave.	Station C % Diff. wrt ave.
α_1	1.39	1.66	-2.32
α_2	-0.10	-0.76	-0.89
α_3	0.70	1.11	-1.28
α_4	-0.50	0.30	0.84

Scaling Factor Percent Differences
α_N wrt α_{Ave} at 23MHz

Scaling Factor	Station A % Diff. wrt ave.	Station B % Diff. wrt ave.	Station C % Diff. wrt ave.
α_1	-0.12	-1.70	-2.78
α_2	1.33	1.29	-0.38
α_3	-1.02	-1.83	-1.02
α_4	0.80	-0.14	0.93

Thus, we have simultaneously quantified both the uncertainty in the displayed dee-voltage and the imbalance between the dee-voltages for the CCP K500 RF System. From the presented data, the extremes of the percent differences in pickup-loop scalings were -2.78 and 1.66. We can therefore say that the uncertainty of any individual dee-voltage is within +/-3% of the displayed dee-voltage. This results in the voltage imbalance between the dee-voltages being within approximately 6% of the displayed dee-voltage. Of course, this is assuming that the system has been tuned such that all loop-voltages are made equal (or the control-room displayed voltages are equal).

The keypoints which can be extracted from the above discussions are:

1.) The control-room display voltages for all dees are based upon the same scaling factor, α_{ave}, which is a function of frequency.
2.) The actual dee-voltage for any individual dee can differ from the displayed voltage by +/-3% of the displayed voltage.
3.) If all displayed voltages are set equal during tuning, the voltage imbalance between the dees can be up to 6% of the displayed voltage value.

The quantitative data presented was based upon a limited number of measurements. Additional measurements can improve the statistics of the data. However, the purpose here was to make an initial step towards the quantification of such measures.

Pickup-Loop Phase Comparisons

The third qualitative measure of the RF system was the uncertainty of the displayed phase-separation between the dees. To quantify this measure, we performed phase-separation measurements between the pickup-loops while the dees were shorted together.

Assuming the short between the dees was an ideal short, the dees themselves were forced to be at the same phase. If in addition we assume that all three resonators were perfectly tuned to the driving frequency, then the phase relationship between the dee-voltage and the induced emf within the pickup-loop would be identical for each resonator. Thus, the phase-separation between all induced emf's should be identically equal to zero.

In the best way possible, we had tried to force the conditions which would satisfy these assumptions. The shorting ring was made as small as possible by being placed close to the dee-tips. It also was made to resemble strap connections so as to minimize the inductance between the connected dees. Furthermore, before the short was made, all three resonators were tuned to the driving frequency such that when the short was put into place all three resonators would be tuned identically. Observations which gave confidence in the assumptions were the following:

1.) When the short was put into place, the system of 3-parallel resonators came up resonant at the driving frequency.
2.) Any mis-tuning of an individual resonator had no discernible affect on the phase-separation between its pickup-loops and an adjacent resonator's pickup-loops

All phase measurements were made relative to pickup-loop DV1 on Station A. The gathered data is presented on the following page. It is broken up into the phase-relationships of a particular loop, DVxx, on each station. This is done since a particular loop, DVxx, is used for the same function from station to station. In particular, pickup-loop DV1 from each station is used for measuring the displayed phase.

Phase Data for DV1 with Dees Shorted

Freq (MHz)	Station A $\phi_{A-DV1} - \phi_{A-DV1}$	Station B $\phi_{B-DV1} - \phi_{A-DV1}$	Station C $\phi_{C-DV1} - \phi_{A-DV1}$
15	0.00	-0.05	-0.10
23	0.00	0.00	0.10

Phase Data for DV2 with Dees Shorted

Freq (MHz)	Station A $\phi_{A-DV2} - \phi_{A-DV1}$	Station B $\phi_{B-DV2} - \phi_{A-DV1}$	Station C $\phi_{C-DV2} - \phi_{A-DV1}$
15	0.00	-0.10	-0.10
23	-0.10	0.10	0.08

Phase Data for DV3 with Dees Shorted

Freq (MHz)	Station A $\phi_{A-DV3} - \phi_{A-DV1}$	Station B $\phi_{B-DV3} - \phi_{A-DV1}$	Station C $\phi_{C-DV3} - \phi_{A-DV1}$
15	0.40	0.50	0.45
23	0.80	0.80	0.80

Phase Data for DV4 with Dees Shorted

Freq (MHz)	Station A $\phi_{A-DV4} - \phi_{A-DV1}$	Station B $\phi_{B-DV4} - \phi_{A-DV1}$	Station C $\phi_{C-DV4} - \phi_{A-DV1}$
15	0.20	0.50	0.60
23	0.80	0.80	0.90

From the data, we can safely say that if the same pickup-loop from each station is used for measuring the resonator-to-resonator phase, the displayed phase between resonators is accurate to within +/-0.10°. In particular, we should use DV1, DV2, or DV3. DV4 appeared to display a 0.3° error from station A to station C.

Conclusion

This note presented investigations of the K500 dee-voltage pickup-loops. In particular the theoretical amplitude scaling factor was compared to the experimental scaling factor. Increasing the ‘effective’ loop area within the theoretical derivation compensated for the discrepancy between theory and experiment (which was found to be a factor of 2.04). Accuracy and station-to-station repeatability investigations revealed the following:

1.) The control-room display voltages for all dees are based upon the same scaling factor, α_{ave}, which is a function of frequency.
2.) The actual dee-voltage for any individual dee can differ from the displayed voltage by +/-3% of the displayed voltage.
3.) If all displayed voltages are set equal during tuning, the voltage imbalance between the dees can be up to 6% of the displayed voltage value.

Phase accuracy investigations revealed the following:

If the same pickup-loop from each station is used for taking resonator-to-resonator phase measurements, the measurement is accurate to within +/-0.10°. However, DV4 appeared to display a 0.3° error from station A to station C, thus we should insist on using either DV1, or DV2, or DV3 as the designated pickup-loop for phase measurements.

The measurements presented here were based upon a novel technique of forcing all three dee-resonators to the same voltage and phase through a dee-tip shorting plate. Although this was an initial attempt with crude justification, the data acquired appears to have placed confidence in the technique.

Appendix A

Theoretical Dee Voltage Pickup-Loop Scaling Factor

The theoretical calculation of the dee-voltage pickup loop scaling factor is based upon the following assumptions:

1.) We have a pure TEM mode at the dee-short
2.) The current at the short is expressed as a pure standing wave

Note, all geometrical derivations are based upon figure A1 below

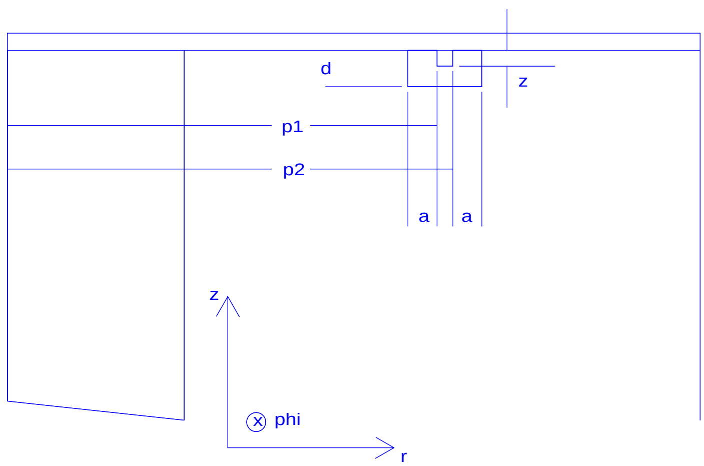


Figure A1: Pickup-Loop Geometry

The governing equation which derives the mathematical relationship between the induced emf and the dee-voltage, is Faraday's law:

$$V_{EMF} = -\frac{d\Phi}{dt} \quad \text{Volts} \qquad (1)$$

where Φ is the magnetic flux intercepted by the loop and V_{EMF}, measured in volts, is the electromotive force induced within the loop.

First, noting that

$$\Phi = \int_S \vec{B} \bullet d\vec{S} \quad \text{Webers (Wb)} \qquad (2),$$

where $\vec{B}$ is the magnetic field at the surface $d\vec{S}$, equation (1) becomes

$$V_{EMF} = \int_S \frac{\partial}{\partial t} \vec{B} \bullet d\vec{S} \qquad (3)$$

Now, assuming there is a pure TEM mode at the short, $\vec{B}$ is related to the current on the short by,

$$\vec{B}(\rho, z) = \frac{\mu}{2\pi\rho} I(t,z)\,\hat{\phi} \quad \text{Wb/m}^2 \qquad (4)$$

where I (Amps) is the flow of positive charge in the $\hat{z}$ direction at location z from the short, ρ is the radial distance from the center axis of the inner conductor, and μ (Henries/meter) is the permeability of air.

The current, $I(z)$, is related to the voltage at the dee by

$$I(t,z) = Y(\omega)\, V_{Dee} \cos(\omega\, t - \frac{\pi}{2}) \cos(\beta\, z) \quad \text{Amps} \quad (5)$$

where $Y(\omega)$ (Amps/Volt) is a frequency-dependent scaling factor relating the current at the short to the voltage at the dee-tip, V_{Dee}, at radian frequency ω as determined from simulations performed in RF Note 116. β (rad/meter) is the propagation constant of air. This equation was derived based upon transmission line equations for a shorted transmission line. The phase term, $\frac{\pi}{2}$, is the phase relationship between the current and the dee-voltage. This is an important factor when considering the phase relationship between the induced emf and the dee-voltage.

Substituting equations (4) and (5) into equation (3) results in

$$V_{EMF} = -\frac{\mu\,\omega}{2\pi} Y(\omega)\, V_{Dee} \sin(\omega\, t - \frac{\pi}{2}) \int_S \frac{1}{\rho} \cos\beta z\, \hat{\phi} \bullet d\vec{S} \qquad (6)$$

Using right-handed conventional notation, $d\vec{S} = d\rho\; dz\; \hat{\phi}$ (m^2). Substituting this into (6) and performing the integration in the region of the loop in figure 1 results in

$$V_{EMF} = -\frac{\mu\,\omega}{2\pi\,\beta}\; Y(\omega)\, V_{Dee} \sin\beta z \ln\left(\frac{\rho_2}{\rho_1}\right) \cos(\omega\, t - \pi) \qquad (7)$$

where ρ_1, ρ_2, and z are the dimensions of the loop area shown in figure 1. The sinusoid term of (6) was replaced with a cosinusoid term in (7) in order to convey the final phase relationship between the induced emf and the dee-voltage. Thus, the induced emf lags the dee-voltage by π radians or 180°. This is an important term when considering phase relationships for the rf control system.

In particular, it is important to note the phase relationship between the induced-emf on the pickup-loop and the signal from the input transmission-line directional coupler. This phase relationship is used as a measure of the resonant frequency of a cyclotron resonator. If the cyclotron tuning drifts, so too will this phase relationship. The drift in tuning is compensated for by moving a capacitive dee-fine tuner. With electrically matched RF cables, the phase relationship between the pickup-loop and the directional-coupler forward signals is determined by using the dee-voltage as a reference point as follows:

1.) The pickup-loop emf lags the dee-voltage by 180°
2.) The transmission line voltage lags the dee-voltage by 90°
3.) The directional-coupler signal lags the transmission line voltage by 90°
4.) Result: the directional-coupler signal lags the dee-voltage by 180° just like the pickup-loop emf. Thus the directional-coupler signal and the pickup-loop emf are in-phase with each other.

Equation (7) can be simplified by noting that for the K500 operating frequency range and the loop dimensions $\beta z << 1$; thus $\sin \beta z$ can be replaced with βz. Equation (7) then reduces to

$$V_{EMF} = -\frac{\mu\omega}{2\pi}\; Y(\omega)V_{Dee}\, z \ln\left(\frac{\rho_2}{\rho_1}\right) \cos(\omega t - \pi) \;\; \text{Volts} \quad (8)$$

Finally, making a change of variables with

$$K_G = z \ln\left(\frac{\rho_2}{\rho_1}\right) \;\text{meters}$$

and substituting $2\pi f$ for ω, equation (8) becomes

$$V_{EMF} = -\mu\, f\, K_G\, Y(f) V_{Dee} \cos(2\pi\, f\, t - \pi) \qquad (9)$$

Equation (9) is the final form which clearly indicates that the relationship involves 2 main proportionality factors, K_G and $Y(f)$, which respectively depend upon the geometry of the loop and the geometry of the resonator. If μ is in Henries/meter, f in Hertz=1/sec., K_G in meters, $Y(f)$ in Amps/Volt, V_{Dee} in Volts, the multiplication results in $\frac{Henries \cdot Amps}{\sec ond}$. Noting that $Henry = \frac{Weber}{Amp}$, the units reduce to Weber/sec which is equivalent to a Volt.

The magnitude of the induced emf is given as

$$\left|V_{EMF}\right| = -\mu\, f\ K_G\ Y(f)\left|V_{Dee}\right| \quad (10)$$

For the discussions of this note, the pickup-loop scaling factor, α , is defined as

$$\alpha = \frac{\left|V_{Dee}\right|}{\left|V_{Loop}\right|} \quad \text{(kVolts/Volt)} \qquad (11)$$

where V_{Loop} is the actual voltage measured with the pickup-loop cable terminated in 50 Ohms. In order to minimize mismatch reflections, a 50 Ohm series source impedance is installed at the actual pickup-loop; thus when the pickup-loop cable is terminated in 50 Ohms an additional factor of 2 is introduced by the voltage divider. Thus, since

$$V_{Loop} = \frac{1}{2}\cdot V_{EMF} \quad (12)$$

the final scaling factor is expressed as

$$\alpha = \frac{2}{1000\cdot \mu\, f\ K_G\ Y(f)} \quad \text{kVolts/Volt} \quad (13)$$

where the factor of 1000 was used to obtain (kVolts/Volt).

Appendix B

X-Ray Calibration Results

Appendix C
Pickup-Loop Measurements & Cable Documentation

Of additional interest for RF system documentation purposes are the phase relationships between the pickup-loops as a function of frequency. This data was accumulated during system tuning data measurements. These measurements involved tuning all resonator systems at frequency intervals across the designed operating range. Once all systems were tuned, phase data was taken. Besides obtaining phase-data for the pickup-loops, phase-data was taken between the directional-coupler forward signal and a dee-voltage pickup loop. Overall, the following phase-data was taken as a function of frequency:

1.) The phase relationships between each station's pickup-loop DVxx with respect to its own DV1 (In this way cable length differences between the pickup-loops can be calculated)
2.) The phase relationship between the transmitter-to-resonator transmission line directional-coupler forward signal and pickup-loop DV1 for each station (This confirms the matching of the directional-coupler forward signal to the pickup-loops for maintaining an adjustable-free resonance control loop. The resonance control loop regulates the tuning of the resonator by adjusting the dee-fine-tuner for a fixed phase between the dee-voltage pickup-loop and the directional-coupler forward signal.)

This Appendix contains plots of the phase vs. frequency for each station's pickup-loops. Included in these plots is the calculated physical length difference (for RG-142 cable) between each pickup-loop cable and the DV1 pickup-loop cable of that particular station; as well as the calculated length differences between the directional-coupler forward signal cable and the DV1 pickup-loop. These calculated length differences can be used for trimming the cables to zero out the phase differences.

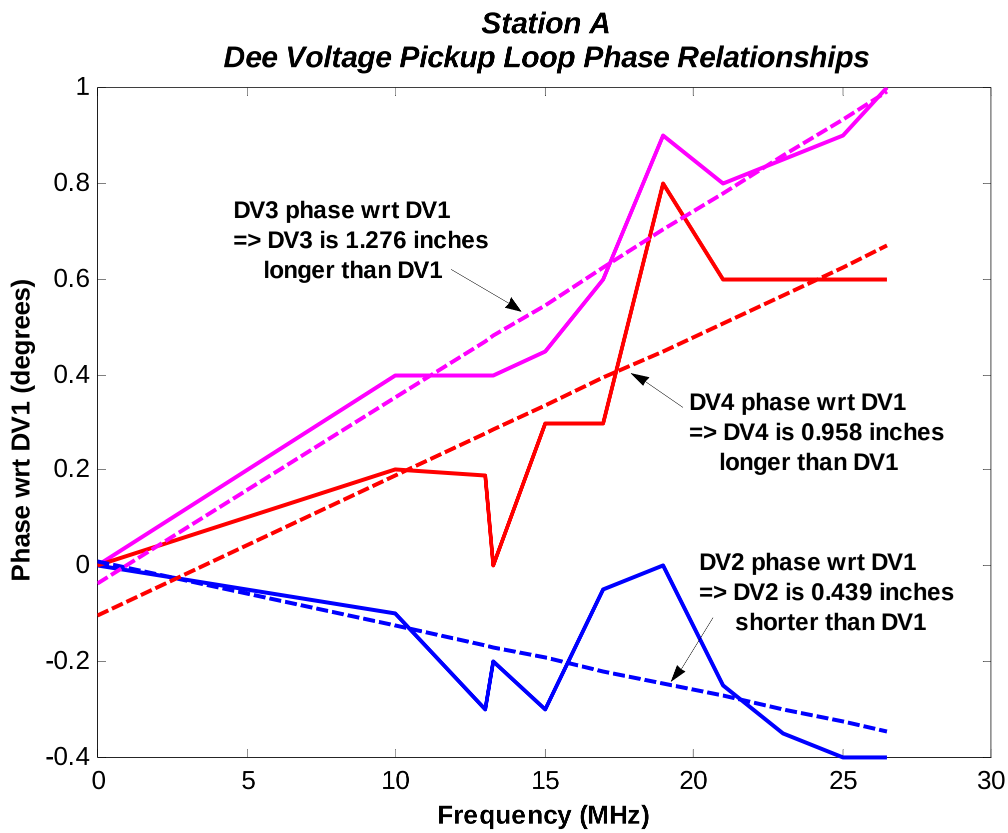
The following table includes the phase vs. frequency data of the directional-coupler with respect to DV1:

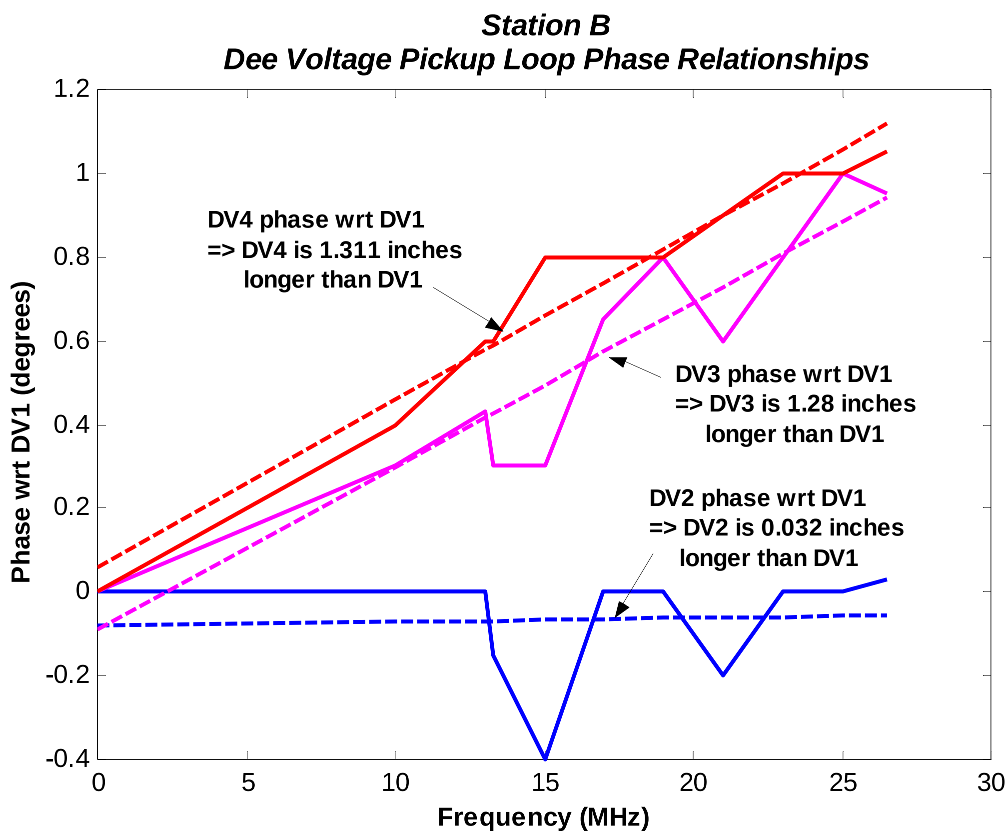
Phase Data for Directional-Coupler Forward Signal

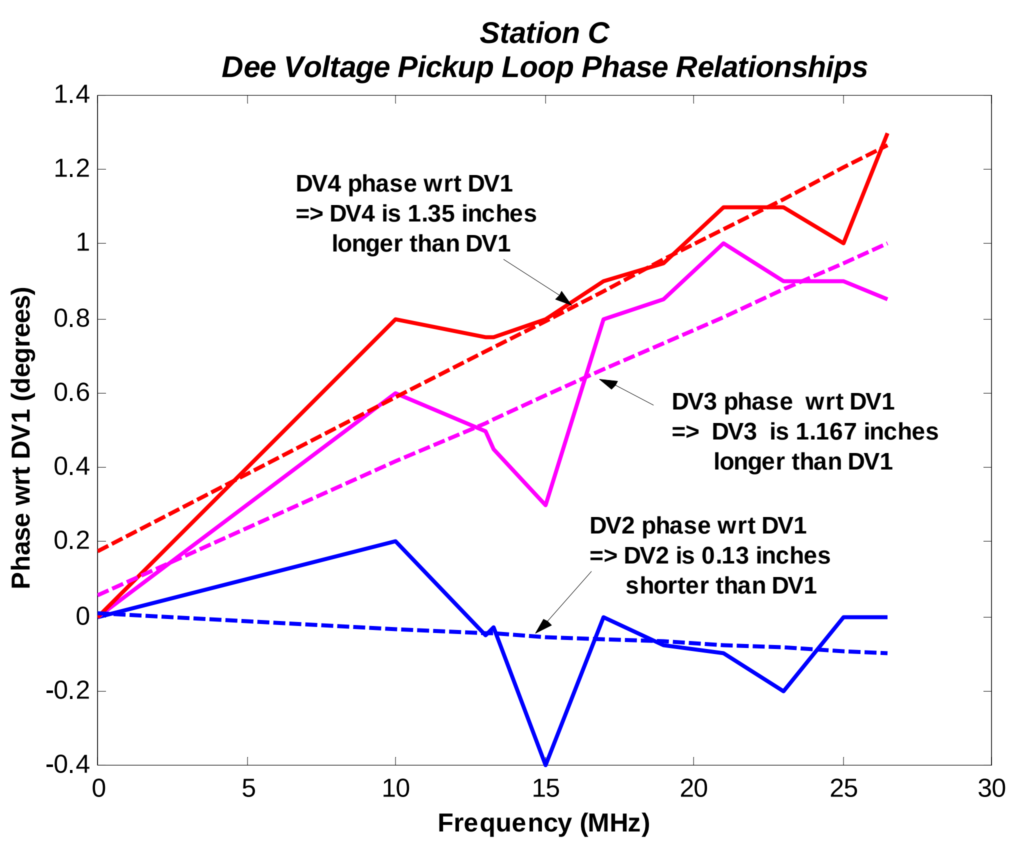
	Station A	Station B	Station C
Freq (MHz)	$\phi_{A\text{-}FWD}$-$\phi_{A\text{-}DV1}$	$\phi_{A\text{-}FWD}$ - $\phi_{A\text{-}DV1}$	$\phi_{A\text{-}FWD}$ - $\phi_{A\text{-}DV1}$
10	10.0	10.5	11.5
13	10.0	13.5	12.0
13.3037	9.9	13.5	13.2
15	12.0	12.0	13.5
17	12.0	15.0	12.0
19	15.0	25.0	18.0
21	15.0	18.0	18.0
23	15.0	18.0	19.0
25	18.0	20.0	19.0
26.5	18.0	16.0	23.0

Plots of the data can be found in this Appendix.

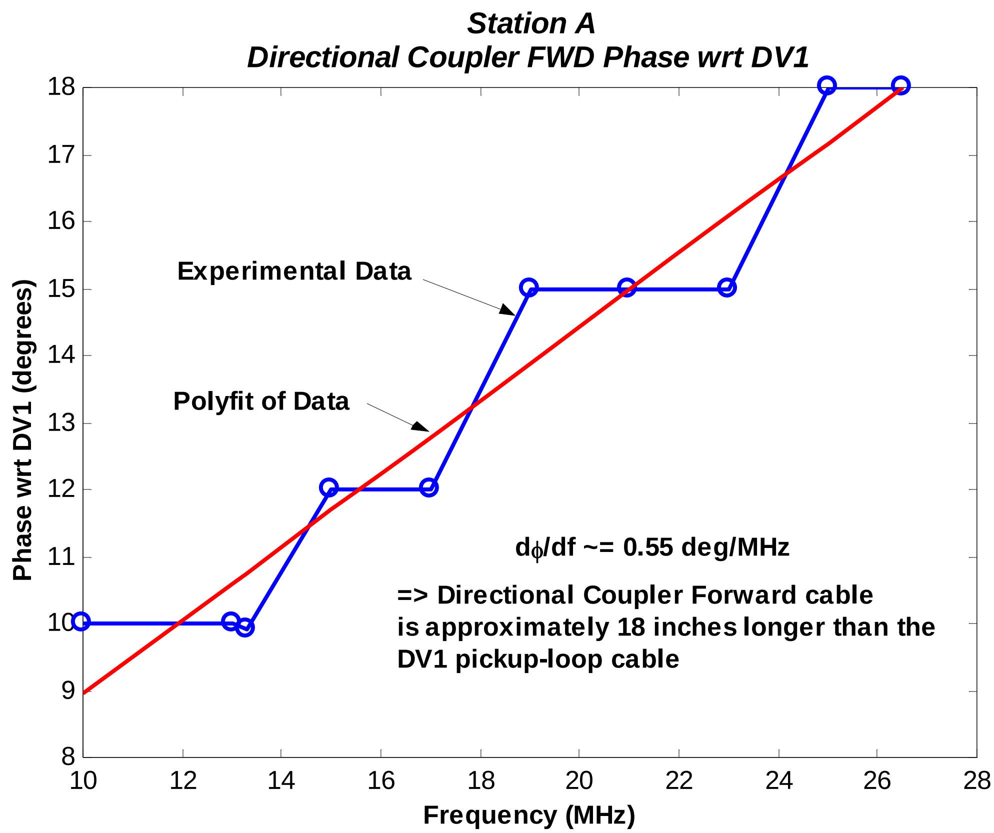
Pickup-Loop Phase Measurements

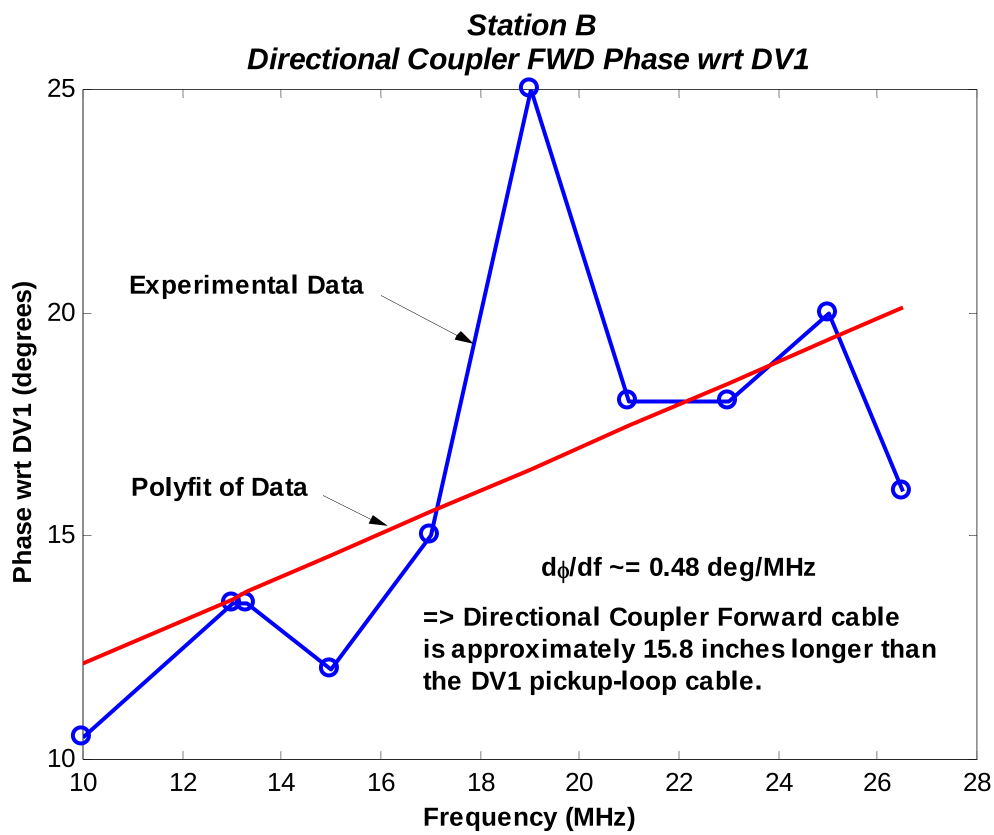


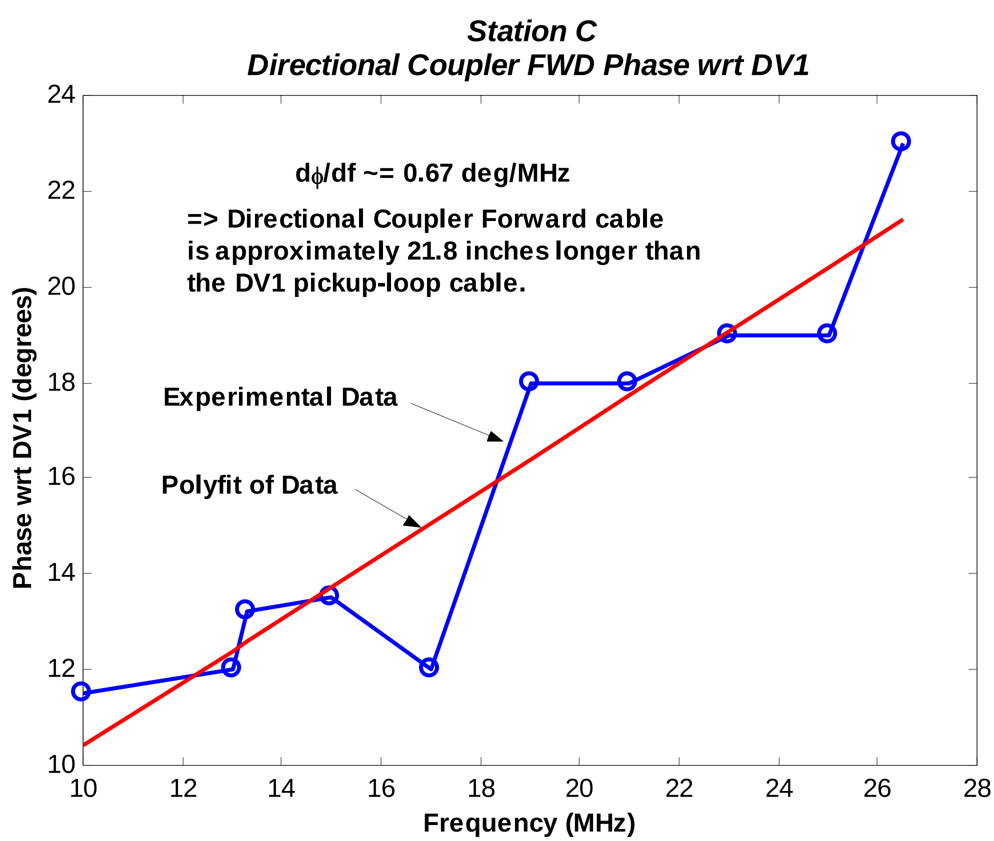

Station B
Dee Voltage Pickup Loop Phase Relationships
Phase wrt DV1 (degrees)
Frequency (MHz)
DV4 phase wrt DV1
=> DV4 is 1.311 inches
longer than DV1
DV3 phase wrt DV1
=> DV3 is 1.28 inches
longer than DV1
DV2 phase wrt DV1
=> DV2 is 0.032 inches
longer than DV1
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
5
10
15
20
25
30


Station C
Dee Voltage Pickup Loop Phase Relationships
Phase wrt DV1 (degrees)
Frequency (MHz)
DV4 phase wrt DV1
=> DV4 is 1.35 inches
longer than DV1
DV3 phase wrt DV1
=> DV3 is 1.167 inches
longer than DV1
DV2 phase wrt DV1
=> DV2 is 0.13 inches
shorter than DV1
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
5
10
15
20
25
30

Directional-Coupler Forward Signal Phase Measurements




Station B
Directional Coupler FWD Phase wrt DV1
Experimental Data
Polyfit of Data
dφ/df ~= 0.48 deg/MHz
=> Directional Coupler Forward cable
is approximately 15.8 inches longer than
the DV1 pickup-loop cable.
Phase wrt DV1 (degrees)
Frequency (MHz)
25
20
15
10
10
12
14
16
18
20
22
24
26
28


Station C
Directional Coupler FWD Phase wrt DV1
dφ/df ~= 0.67 deg/MHz
=> Directional Coupler Forward cable
is approximately 21.8 inches longer than
the DV1 pickup-loop cable.
Experimental Data
Polyfit of Data
Phase wrt DV1 (degrees)
Frequency (MHz)
24
22
20
18
16
14
12
10
10
12
14
16
18
20
22
24
26
28